Orientalional Analysis of the Vesic's Bearing Capacity of Shallow Foundations

L. F. dos Santos, A.C.de Freitas

Abstract. The determination of the bearing capacity of shallow foundations can be considered as a complex elasto-plastic deformation problem, which is often studied phenomenologically. The phenomenological equations are naturally constructed based on the typical dimensions of the variables involved (unit weight, foundation size, etc.) – all physical laws must guarantee the principle of dimensional balance. Nevertheless, there is another requirement physical laws must obey that is less often explored: all equations must be orientationally balanced. While this requirement is obvious for vectorial equations, the laws of continuum mechanics often mix tensors, vectors and scalar quantities. Moreover, not all scalar quantities are considered orientationless (for instance, areas and angles define an orientation determined by the unit vector normal to the plane). Here, the main equations found in the literature for the bearing capacity of soils will be analyzed, testing the orientational balance of the phenomenological equations. It will be shown that not all equations are well balanced – in particular the critical rigidity index for the generalized failure of soils determined by Vesic (1973) is not balanced. Perhaps not surprisingly, the equations that are well balanced lead to a good agreement with experimental data from the literature, while the critical rigidity index fails systematically when compared to tests in model foundations on sand.

Keywords: bearing capacity, critical rigidity index, orientational balance, shallow foundations.

1. Introduction

Analyzing the variables that govern a physical phenomenon, one may extract (at least partially) the mathematical relations between these variables. This is the principle behind the dimensional analysis, which provides the means to determine physical laws even before any analytical or phenomenological derivation is attempted. A complementary approach is provided by the orientational analysis.

This method is based upon assigning orientational symbols to physical quantities such as force, velocity and position, which are spatially oriented. While the orientational symbols are evident for these vector quantities, they may become rather intricate for tensorial quantities or even for some scalars, which may identify an orientation. These symbols sustain a multiplication rule that forms a noncyclic Abelian group with four members, and they can be used to derive additional information that resolves problems incompletely solved by conventional dimensional analysis.

The goal of this work is to provide the orientational analysis of the equations for the bearing capacity (Vesic, 1975), as well as the orientational analysis of the rigidity index I_r and the critical rigidity index $I_{r\text{crit}}$ which are proposed by Vesic for determining if a soil-foundation system will fail in a generalized or non-generalized mode.

The limit stress to generate soil rupture is called Terzaghi's equation (1943) for foundation shapes with $L \gg B$ (Vesic (1973), as shown in Fig. 1, has already proposed a shape correction factor for different foundation forms of $L \gg B$), given by:

$$q_{rup} = c \cdot N_c + q \cdot N_q + \frac{1}{2} \gamma \cdot B \cdot N_\gamma \quad (1)$$

where c is the soil cohesion intercept, q is the uniformly distributed load due to the overburden, γ is the soil unit weight, B is the smallest size of the foundation base and the N_c , N_q and N_γ are dimensionless bearing capacity factors, defined by:

$$N_c = (N_q - 1) \cot \phi \quad (2)$$

$$N_q = e^{(\pi \tan \phi)} \left(\tan \left(\frac{\pi}{4} + \frac{\phi}{2} \right) \right)^2 \quad (3)$$

$$N_\gamma = 2(N_q + 1) \tan \phi \quad (4)$$

These equations are stated for lower compressibility soils, *i.e.*, idealized soil-foundation systems that fail as a rigid-plastic medium, instead of elasto-plastic. This idealization is considered by Vesic (1975) to be adequate as long as the rigidity index (Eq. 5) is larger than a critical rigidity index (Eq. 6).

Luiza Fernandes dos Santos, M.Sc. Researcher, Programa de Projeto de Estruturas, Escola Politécnica, Universidade Federal do Rio de Janeiro, Av. Athos da Silveira Ramos 149, Ilha do Fundão, Centro de Tecnologia - Bloco A, 2º andar, Cidade Universitária, Rio de Janeiro, RJ, Brazil. e-mail: luizadosfs@gmail.com.

Alessandra Conde de Freitas, D.Sc., Associate Professor, Programa de Projeto de Estruturas, Escola Politécnica, Universidade Federal do Rio de Janeiro, Av. Athos da Silveira Ramos 149, Ilha do Fundão, Centro de Tecnologia - Bloco A, 2º andar, Cidade Universitária, Rio de Janeiro, RJ, Brazil. e-mail: alessandracfreitas@poli.ufrj.br.

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Figure 1 - Shallow foundation supported on a soil without overburden.

$$I_r = \frac{G}{c + \sigma_{med} \tan \phi} \quad (5)$$

$$I_{r \text{ critical}} = \frac{1}{2} e^{(3.30 - 0.45 \frac{B}{L}) \cot(\frac{\pi - \phi}{4})} \quad (6)$$

where G is the shear modulus, ϕ is soil friction angle and σ_{med} is a characteristic stress scale, estimated as the mean stress at a depth $B/2$ below the base of the footing. The shear modulus and mean stress can be defined respectively as:

$$G = \frac{E}{2(1 + \nu)} \quad (7)$$

$$\sigma_{med} = \frac{\sigma'_v + 2\sigma'_h}{3} \quad (8)$$

where E is Young's modulus, ν is the Poisson ratio and σ_v and σ_h are the effective vertical and horizontal stresses at a depth $B/2$ below the base of the footing, respectively. The expressions for σ_v and σ_h are respectively:

$$\sigma'_v = \frac{1}{2} B \cdot \gamma \quad (9)$$

$$\sigma'_h = \sigma'_v \cdot k_0 \quad (10)$$

where k_0 is the at-rest earth pressure coefficient and it is defined as

$$k_0 = 1 - \sin \phi \quad (11)$$

The Eq. 11 applies only for sands.

2. Material and Methods

2.1. Orientational analysis in general

The symbol $\doteq$ will denote an orientational equality (Siano, 1985a), *i.e.*, two quantities that carry the same orientational symbol will be related by the symbol $\doteq$. The symbols for orientations in the Cartesian directions x , y and z are respectively l_x , l_y and l_z . Equivalent symbols and their multiplication table may be found for other coordinate systems (Siano, 1985b), but will not be necessary for our purpose here. Quantities without an orientation [some scalars

(mass, time, ...) and tensor elements (normal stress, normal strain, ...)] will be denoted by the symbol l_0 (identity). To understand why quantities such as normal strain are deemed orientationless, the multiplication table for these symbols will be shown now.

The product between two quantities with different orientations will also have an orientation, following the respective multiplication table, which is analogous to the vector product rule, but commutative (without taking the signal into account), *i.e.*,

$$l_y \cdot l_z \doteq l_x \doteq l_z \cdot l_y \quad (12)$$

$$l_x \cdot l_z \doteq l_y \doteq l_z \cdot l_x \quad (13)$$

$$l_x \cdot l_y \doteq l_z \doteq l_y \cdot l_x \quad (14)$$

There is also a rule of multiplication between two oriented quantities with same orientation, which always generates a quantity without orientation (l_0):

$$l_x \cdot l_x \doteq l_0 \quad (15)$$

$$l_y \cdot l_y \doteq l_0 \quad (16)$$

$$l_z \cdot l_z \doteq l_0 \quad (17)$$

$$l_0 \cdot l_0 \doteq l_0 \quad (18)$$

The identity of this multiplication operation is l_0 , such that multiplying it by any orientational symbol generates the same orientation:

$$l_x \cdot l_0 \doteq l_x \doteq l_0 \cdot l_x \quad (19)$$

$$l_y \cdot l_0 \doteq l_y \doteq l_0 \cdot l_y \quad (20)$$

$$l_z \cdot l_0 \doteq l_z \doteq l_0 \cdot l_z \quad (21)$$

And finally, every symbol of orientation is the inverse of itself, that is,

$$l_0^{-1} \doteq l_0 \quad (22)$$

$$l_x^{-1} \doteq l_x \quad (23)$$

$$l_y^{-1} \doteq l_y \quad (24)$$

$$l_z^{-1} \doteq l_z \quad (25)$$

It is possible to summarize all this in a single matrix (Fig. 2).

Therefore, these symbols and this multiplication operation form an Abelian group.

There is no definition of fractional powers of orientational symbols, so one may not have rational exponents in the original equation (physical law) when applying orientational analysis. In order to analyze the directionality of an equation which contains some root (fractional exponent), it is necessary to eliminate it by exponentiation.

For example, the orientational symbol of the area of the square A is l_z , since

$$A = L^2 \doteq l_x \cdot l_y \doteq l_z \doteq A \quad (26)$$

	l_0	l_x	l_y	l_z
l_0	l_0	l_x	l_y	l_z
l_x	l_x	l_0	l_z	l_y
l_y	l_y	l_z	l_0	l_x
l_z	l_z	l_y	l_x	l_0

Figure 2 - General orientational matrix.

This also illustrates how a scalar quantity may bear an orientation symbol. In this example, the symbol is carried by the area of a square, which is a natural notion when dealing with the calculus of multivariate fields (for instance, the calculation of a flux in Gauss' theorem is dependent on the orientation of the area of the Gaussian surface of choice).

The fractional power would also be eliminated if the equation of the area represented as fractional power ($L = \sqrt{A}$) were raised to the fourth power:

$$(L)^4 = (\sqrt{A})^4 \quad (27)$$

$$L^4 = A^2 \quad (28)$$

But this choice of exponent makes the orientational analysis less effective. Eqs. 15-18 show that even powers of any quantity will lead to the trivial conclusion $l_0 \doteq l_0$. The form of Eq. 26 is referenced as the normal form, highlighting its usefulness for orientation analyses.

It can be concluded that the orientation of unknown quantities can be identified by balancing the orientational symbols derived from the physical law that defines this quantity.

2.2. Orientational analysis of angles

Angles are dimensionless, but they do carry an orientational symbol, so that equations associating vectors in different directions become directionally homogeneous, which is an important characteristic of orientational analysis.

An example of orientational angle analysis is found in the simple problem of the dynamics of a body in a tilted plane (Fig. 3).

Balancing the forces on the x and y axes leads to the conclusion that the friction coefficient is given by $\mu = \tan \theta$. The orientational analysis of the Amontons first law gives

$$F_{at} = \mu \cdot N \quad (29)$$

$$l_x \doteq l_\mu \cdot l_y \quad (30)$$

Therefore, by homogeneity,

$$l_\mu \doteq l_x \cdot l_y^{-1} \doteq l_z \quad (31)$$

with this, it can be concluded that $\tan \theta$ has direction l_z .

Figure 3 - Schematic of an inclined plane block.

3. Results

3.1. Orientational analysis of bearing capacity factors for the Terzaghi's equation

When analyzing Eq. 1, it can be verified that N_c , N_q and N_γ are adimensional:

$$[q_{rup}] = [c] \cdot [N_c] + [q] \cdot [N_q] + \frac{1}{2} [\gamma] \cdot [B] \cdot [N_\gamma] \quad (32)$$

$$\left[\frac{M}{LT^2} \right] = \left[\frac{M}{LT^2} \right] \cdot [N_c] + \left[\frac{M}{LT^2} \right] \cdot [N_q] + \left[\frac{M}{LT^2} \right] \cdot [N_\gamma]$$

Now, the orientational balance of the equations discussed in Siano (1985a) for N_c , N_q and N_γ (Eq. 2-4) will be analyzed. Since all three equations contain $\tan(\phi)$, and N_γ contains $(\tan(\frac{\pi}{4} + \frac{\phi}{2}))^2$, what will be discussed is how to obtain the orientational analysis of these functions. While the tangent function was already discussed, it is useful for other purposes to obtain its orientational symbol by analyzing its Taylor's series expansion. For both expressions, it can be written as:

$$\tan \phi = \phi + \frac{\phi^3}{3} + \frac{2\phi^5}{15} + \dots \quad (33)$$

$$\left(\tan \left(\frac{\pi}{4} + \frac{\phi}{2} \right) \right)^2 = 1 + 2\phi + 2\phi^2 + \frac{5\phi^3}{3} + \frac{4\phi^4}{3} + \frac{61\phi^5}{60} + \dots \quad (34)$$

If angles had a dimension ascribed to them, this expression would make no sense, since it would add distinct powers (for example, $\phi^3 + \phi^5$). However, they can bear an orientational symbol. Indeed, since all powers in the Taylor expansion are odd, the orientation of the tangent function is the same as that of the friction angle ϕ . Notice that Eq. 34 involves the square of a tangent function, which is orientationless. Indeed, its Taylor series expansion contains even and odd powers of ϕ , and it cannot be balanced unless each term is orientationless (notice that the series terms containing odd powers of ϕ may still be orientationless, as long as the series coefficient has orientation).

Functions with no definite parity (which are neither even nor odd) need to have both dimensionless and directionless arguments. For example, e^x

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \dots \quad (35)$$

If $x \doteq l_z$, it can be concluded that the orientational balance of Eq. 35 is:

$$l_{e^x} \doteq l_0 + l_z + \frac{l_z^2}{2} + \frac{l_z^3}{6} + \dots \quad (36)$$

$$l_{e^x} \doteq l_0 + l_z + l_0 + l_z + \dots \quad (37)$$

which is not directionally balanced. Therefore,

$$e^x \doteq l_0, \text{ for } x \doteq l_0 \quad (38)$$

Frequently, an apparently orientational exponent is rendered orientationless by a pre-factor with a directionality that must be taken into account. This prefactor often contains π (3.1415 ...), which is observed in the expression for N_q , since $e^{(\pi \tan \phi)}$ would violate the orientational balance, if π had no direction. Because $\tan \phi \doteq l_z$, its can be concluded that $\pi \doteq l_z$:

$$\pi \tan \phi \doteq l_z \cdot l_z \doteq l_0 \quad (39)$$

and therefore

$$e^{(\pi \tan \phi)} \doteq l_0 \quad (40)$$

The whole expression for N_q is therefore balanced as

$$N_q = e^{(\pi \tan \phi)} \left(\tan \left(\frac{\pi}{4} + \frac{\phi}{2} \right) \right)^2 \quad (41)$$

The balance of N_γ is also obtained as

$$\begin{aligned} N_\gamma &= 2(N_q + 1) \tan(\phi) \\ N_\gamma &\doteq l_0 \cdot (l_0 + l_0) \cdot l_z \doteq l_z \end{aligned} \quad (42)$$

And finally, orientational analysis of N_c gives

$$\begin{aligned} N_c &= (N_q - 1) \cot(\phi) \\ N_c &\doteq (l_0 - l_0) \cdot l_z^{-1} \doteq l_0 \cdot l_z \doteq l_z \end{aligned} \quad (43)$$

The orientational balance of Eq. 1 will be checked by determining the orientational symbol for q_{rup} , q , c , γ and B . The coordinate system in Fig. 4 will be adopted, with the load applied vertically along the y axis.

Some symbols are readily recognized as:

$$\sigma = \frac{P}{A} \doteq \frac{l_y}{l_x \cdot l_z} \doteq \frac{l_y}{l_y} \doteq l_0 \quad (44)$$

$$q_{rup} = \frac{\text{weight of structure}}{\text{Area}} \doteq \frac{l_y}{l_x \cdot l_z} \doteq \frac{l_y}{l_y} \doteq l_0 \quad (45)$$

Figure 4 - Stress components in the Cartesian coordinate system.

$$q = \frac{\text{weight of overburden}}{\text{Area}} \doteq \frac{l_y}{l_x \cdot l_z} \doteq \frac{l_y}{l_y} \doteq l_0 \quad (46)$$

$$\gamma = \frac{\text{weight of sand mas}}{\text{Volume}} \doteq \frac{l_y}{l_x \cdot l_y \cdot l_z} \doteq \frac{l_y}{l_0} \doteq l_y \quad (47)$$

$$\begin{aligned} B &\doteq l_y \\ \sigma &= \frac{P}{A} \doteq \frac{l_y}{l_x \cdot l_z} \doteq \frac{l_y}{l_y} \doteq l_0 \end{aligned} \quad (48)$$

To determine the orientational symbol of c , its definition will be used in terms of the shear failure envelope curve, given by $\tau = c + \sigma \cdot \tan \phi$. This gives:

$$\tau = c + \sigma \tan \phi \doteq c + l_0 \cdot l_z \doteq c + l_z \quad (49)$$

which is balanced only if c has orientation l_z . Then, it is obtained by dimensional analysis of Eq. 1:

$$\begin{aligned} q_{rup} &= c \cdot N_c + q \cdot N_q + \frac{1}{2} \gamma \cdot B \cdot N_\gamma \\ l_0 &\doteq l_z \cdot N_c + l_0 \cdot N_q + l_0 \cdot l_y \cdot l_x \cdot N_\gamma \\ l_0 &\doteq l_z \cdot N_c + l_0 \cdot N_q + l_z \cdot N_\gamma \end{aligned} \quad (50)$$

with this, it is concluded that the orientational symbol of N_q , N_γ and N_c should be respectively l_0 , l_z and l_z , in agreement with the results in Eqs. 41, 42 and 43. So Eq. 1 is directionally balanced.

3.2. Orientational analysis of the Vesic equation for determination of generalized and non-generalized rupture

As already shown in Eq. 5, the rigidity index, proposed by Vesic (1973), is given by:

$$I_r = \frac{G}{c + \sigma_{med} \tan \phi}$$

First, a orientational analysis of the Elasticity Theory equations must be performed, especially the relationship between stresses and strains. At first, normal strain does not carry a directionality

$$\varepsilon_x = \frac{\Delta L_x}{L_x} \doteq \frac{l_x}{l_x} \doteq l_0 \quad (51)$$

But ε_{ij} is a tensor. In fact, for tensors, the diagonal elements have no direction, so that $\varepsilon_{xx} \doteq l_0$ is to be expected.

This helps to understand the direction of the elastic modules. Hooke's law can be expressed in three dimensions, where each normal strain is related to the three components of normal stress by the material properties, E and ν :

$$\varepsilon_x = \frac{1}{E} [\sigma_x - \nu(\sigma_y + \sigma_z)] \quad (52)$$

$$\varepsilon_y = \frac{1}{E} [\sigma_y - \nu(\sigma_x + \sigma_z)] \quad (53)$$

$$\varepsilon_z = \frac{1}{E} [\sigma_z - \nu(\sigma_x + \sigma_y)] \quad (54)$$

Since the orientational symbols of $\sigma_x, \sigma_y, \sigma_z, \varepsilon_x, \varepsilon_y, \varepsilon_z$, are l_0 , as shown in Fig. 5 and Eq. 51, then by balancing it can be seen that E and ν are also l_0 . With this, it can be concluded that G , shear modulus, also has direction l_0 , since:

$$G = \frac{E}{2(1+\nu)} \quad (55)$$

$$G \doteq \frac{l_0}{2(1+l_0)} \doteq l_0$$

Thus, the rigidity index (Eq. 5) has direction l_z , since:

$$I_r = \frac{G}{c + \sigma_{med} \cdot \tan \phi} \quad (56)$$

$$I_r \doteq \frac{l_0}{l_z + l_0 \cdot l_z} \doteq \frac{l_0}{l_z + l_z} \doteq \frac{l_0}{l_z} \doteq l_0 \cdot l_z^{-1} \doteq l_z$$

In order to be comparable to I_r , the critical rigidity index of soil $I_{r\text{crit}}$ should be orientationally consistent, *i.e.*, it should carry an orientational symbol l_z . However, when analyzing the orientational balance of this equation, a orientational inconsistency is found. According to Eq. 6, the critical rigidity index, for the cases of drained sand analysis, is given by:

$$I_{r\text{critical}} = \frac{1}{2} e^{(3.30-0.45 \frac{B}{L}) \cot(\frac{\pi}{4} - \frac{\phi}{2})}$$

$$\begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{pmatrix} \doteq \begin{pmatrix} l_0 & l_z & l_y \\ l_z & l_0 & l_x \\ l_y & l_x & l_0 \end{pmatrix}$$

Figure 5 - Strain matrix and orientational deformation matrix.

Keeping the axes as in Fig. 4, the directions of the length and width of the foundation base are respectively $l_B \doteq l_x$ and $l_L \doteq l_z$.

As demonstrated previously in Eq. 38, $e^x \doteq l_0$, for $x \doteq l_0$, and the exponent cannot be directionally non-trivial. Therefore $(3.30-0.45 \frac{B}{L}) \cot(\frac{\pi}{4} - \frac{\phi}{2})$ should be l_0 . But what happens is:

$$\begin{aligned} \left(3.30-0.45 \frac{B}{L}\right) \cot\left(\frac{\pi}{4}-\frac{\phi}{2}\right) &\doteq \left(l_0-l_0 \cdot \frac{l_x}{l_z}\right) \cdot l_{\cot\left(\frac{\pi}{4}-\frac{\phi}{2}\right)} \\ \left(3.30-0.45 \frac{B}{L}\right) \cot\left(\frac{\pi}{4}-\frac{\phi}{2}\right) &\doteq (l_0-l_0 \cdot l_y) \cdot l_{\cot\left(\frac{\pi}{4}-\frac{\phi}{2}\right)} \\ \left(3.30-0.45 \frac{B}{L}\right) \cot\left(\frac{\pi}{4}-\frac{\phi}{2}\right) &\doteq (l_0-l_y) \cdot l_{\cot\left(\frac{\pi}{4}-\frac{\phi}{2}\right)} \end{aligned} \quad (57)$$

The operation $\left(3.30-0.45 \frac{B}{L}\right) \cot\left(\frac{\pi}{4}-\frac{\phi}{2}\right)$ on the right-hand side of Eq. 57 is inconsistent with orientational balancing, and therefore there must be something unphysical about this empirical critical rigidity index.

4. Conclusions

One may conclude that bearing capacity factors are directionally consistent. Often, a new proposal for a more accurate estimation of the bearing capacity factors N_c, N_q and N_γ is presented in the literature (Michalowski, 1997). The analysis presented here may provide a fast consistency check for these expressions. For instance, N_c and N_γ must be odd functions of the friction angle ϕ , while N_q must be even.

Although a orientational balance does not guarantee the accuracy of the equation, something may be stated about an unbalanced equation. For instance, the rigidity index equation for determining the limit between a generalized and a non-generalized rupture is not balanced. Vesic, in his article entitled "Bearing Capacity of Deep Foundations in Sand" (Vesic, 1963), shows experimental results in tests with circular and rectangular plates on the surface of the sandy soil. In each test, the rupture is classified as generalized, local or punching.

In Table 1, the experimental results are compared with the predictions from the analytical equation of critical rigidity index. The results shown in bold do not match those obtained experimentally. With this, it can be concluded that the criterion of Vesic is inaccurate in this regime. Furthermore, the injudicious use of this criterion jeopardizes the safety of the foundations, since a prediction of generalized failure when the soil is actually well-compressible may result in an overestimated ultimate load capacity (disregarding the necessary correction factor). The inadequacy of this equation could be anticipated by the orientational analysis of the proposed equation, which clearly raises doubts about this expression.

Table 1 - Experimental results (Vesic, 1963) and the analytical result (Vesic, 1975).

Test #	Experimental results [Vesic (1963)]				Rigidity Index criterion [Vesic (1975)]			
	B (m)	γ_d (kN/m ³)	ϕ (°)	Failure	E (kN/m ²)	$(I_r)_{crit}$	I_r	Failure
34	0.05	15.37	43.15	General	1,821.75	359.58	3,463.46	General
21		14.90	41.09	Local	1,820.36	263.39	3,681.14	General
22		14.38	38.85	Local	1,820.43	192.85	3,934.37	General
23		13.23	34.10	Punching	1,811.57	107.58	4,581.91	General
44	0.10	15.50	43.72	General	2,476.09	393.74	2,506.39	General
41		15.02	41.61	Local	2,476.26	284.25	2,665.09	General
42		14.58	39.70	Local	2,479.70	216.41	2,815.84	General
43		13.34	34.54	Punching	2,464.05	113.11	3,317.25	General
61	0.15	15.41	43.33	General	3,057.14	369.95	2,053.11	General
62		14.89	41.04	Local	3,059.30	261.49	2,193.6	General
63		14.69	40.18	Local	3,059.14	231.33	2,250.15	General
64		15.22	42.49	Punching	3,058.21	324.55	2,103.16	General
84	0.20	15.41	43.33	General	3,530.08	369.95	1,778.05	General
81		15.25	42.62	General	3,531.21	331.10	1,814.55	General
82		15.25	42.62	General	3,531.21	331.10	1,814.55	General
83		14.09	37.63	Local	3,528.89	164.46	2,105.97	General
16	rectangular 0.051 x 0.30	15.44	43.46	General	1,764.97	903.27	3,542.85	General
1		14.99	41.48	Local	1,766.13	640.82	3,750.94	General
2		14.72	40.31	Local	1,766.21	529.55	3,882.37	General
3		13.45	34.98	Punching	1,759.65	244.54	4,586.13	General

In Civil Engineering problems, the power of orientational analysis, unlike dimensional analysis, is not yet well explored and can lead to important advances. The mathematical structure of the laws of elasticity and plasticity allows their broad application and can guide efforts in determining appropriate empirical equations for quantities of interest.

The elastic modulus E was not provided in Vesic (1963), but an expression is reported in Vesic (1973) associating this modulus to the mean normal stress $E = E_1 \sqrt{\sigma_m / \sigma_1}$, with $E_1 = 39,180.65 \text{ kN/m}^2$ (364 ton/ft²) being the modulus at mean normal stress of $\sigma_1 = 104.64 \text{ kN/m}^2$ (1 ton/ft²).

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List of Symbols

- B - smaller length of shallow foundations
 c - soil cohesion intercept
 E - Young's modulus
 G - shear modulus
 I_r - rigidity index
 $I_{r,critical}$ - critical rigidity index
 k_0 - coefficient of earth pressure at rest
 L - longer length of shallow foundations
 N_c, N_q, N_γ - bearing capacity factors
 q - uniformly distributed load
 q_{rup} - rupture stress/ ultimate pressure
 ε - normal strain

ϕ - friction angle

γ - unit weight

γ_d - unit weight of the dry soil

ν - Poisson ratio

σ_h - horizontal stress

σ_{med} - mean stress at a depth $B/2$ below the base of the footing

σ_v - vertical stress