Abstract. Due to the complexity and uncertainty involved in mass movements, it is not possible to predict their occurrence with accuracy. This uncertainty is due to the space-time variability of physical soil parameters and processes, which determine the boundary conditions of the problem. In this study, we analyzed the influence of spatial variability of the parameters that determine soil resistance, the water retention curve and the unsaturated hydraulic conductivity function on the Factor of Safety of a hypothetical slope. To determine the parameters to which the slope is more sensitive, we carried out a sensitivity analysis using the First-Order Second-Moment Method. We assumed that spatial variability follows normal and lognormal distribution, by using a methodology based on Monte Carlo Simulations and a Kriging process. The water flow equation is solved using numerical methods and the Factor of Safety was found with the Limit Equilibrium Method. According to the sensitivity analysis, the parameters that most affect the stability of the slope analyzed are cohesion, friction angle, air-entry value, and saturated hydraulic conductivity. When varying the air-entry value and cohesion, the probability distributions have very low dispersion, and mode has values similar to the deterministic values of Factor of Safety. The hydraulic conductivity variation results in that the values of mode move further away from the deterministic Factor of Safety as time progresses, increasing at the same time the dispersion. When all parameters are varied simultaneously, the behavior of the Factor of Safety is highly influenced by the hydraulic conductivity.

Keywords: factor of safety (FS), hydrology, landslides, uncertainty, unsaturated soil.

1. Introduction

Landslides are among the most frequent natural disasters in the world, especially in tropical regions, which are characterized by having deep and weathered soil profiles, steep slopes, and high precipitation (Cho, 2014). Understanding the mechanical behavior of soil-water system becomes relevant since landslides cause significant human and economic losses (Tarlolli et al., 2011; Legorreta Paulin et al., 2016).

Due to the complexity and uncertainty surrounding the landslide phenomenon, it is not possible to accurately predict its occurrence (Metry & Bhattacharya, 2015). Uncertainty could be the result of many factors, such as, the natural spatial and temporal variability of geotechnical and hydrological parameters (Cho, 2007; Casagli et al., 2009; Imaizumi et al., 2009), limitations of in-situ research, and imperfections in the applied model (Lacasse, 2013; Raia et al., 2013; Athapaththu et al., 2015). However, estimations of slope safety level are often based upon deterministic values that result in unrealistic hazard assessments, being necessary to develop tools that allow the inclusion of the effects of probability.

The most common probabilistic approaches used in geotechnical engineering are namely: Point Estimate Method, PEM, First-Order Second-Moment Method, FOSM, and Monte-Carlo Simulation Method, MSM, (e.g. Chowdhury 1986; Christian et al., 1992; Chowdhury 1992; Duncan 2000; Griffiths & Fenton 2004; Chok 2008; Griffiths et al., 2009).

The spatial variability of soil properties has been stochastically studied since Alonso (1976), who showed the uncertainty level of the parameters involved and the degree of correlation of soil properties. Li & Lumb (1987), Cho (2007), Wang et al. (2011), and Li et al. (2013) studied spatial variability under the framework of the Limit Equilibrium Method, FEM. Li & Lumb (1987) discussed some improvements on the FOSM probabilistic approach to slope design. Cho (2007) and Wang et al. (2011) presented a numerical procedure for a probabilistic slope stability analysis based on MSM. Other methods used to study the spatial variability of soil parameters are the Random Element Method, RFEM, (e.g., Griffiths & Fenton 2004; Chok 2008; Griffiths et al., 2009; Hue Le et al., 2015), and the Stochastic Finite Element Method, SFEM, (e.g., Farah et al., 2011; Jiang et al., 2014).

Among parameters frequently studied are: hydraulic conductivity (e.g., Griffiths & Fenton 1993; Gui et al., 2000; Srivastava et al., 2010; Santos et al., 2011; Otávalo & Cordão-Neto 2013), cohesion (e.g., Li & Lumb 1987; Cho 2007; Li et al., 2013; Tietje et al., 2014), friction angle (e.g., Cho 2007; Griffiths et al., 2009; Jiang et al., 2014);
Kriging process is used to interpolate the values of each parameter in the nodes and calculate the value in the element, while the MSM is employed to find the probability density function of Factor of Safety, FS.

In this analysis, we propose a methodology to assess the influence of spatial variability on unsaturated slope stability based on the MSM and a Kriging process. The Kriging process is used to interpolate the values of each parameter in the nodes and calculate the value in the element, while the MSM is employed to find the probability density function of Factor of Safety, FS.

We use the Finite Elements Method, FEM, and Finite Difference Method, FDM, to solve the water flow equation numerically, and the LEM to find the Factor of Safety, FS. The varied parameters are determined through a sensitivity analysis with the FOSM. This methodology allows the varying of soil parameters using the same equations to solve in space the flow equation numerically, i.e., the shape functions.

2. Flow and Stability Analysis

When assessing slope stability, the processes occurring within the soil mass due to changes in water content must be simulated, which is possible by solving the unsaturated soil flow equation. In this analysis, the spatial and temporal solutions are determined using the FEM and FDM, respectively.

As the flow equation solution is a function of the hydraulic head, h, it is necessary to define constitutive models of hydraulic conductivity and water content volume involving h. As water content constitutive relation, we use the Soil Water Retention Curve, SWRC, by Van Genuchten (1980), expressed by

$$
\Theta = \left[ \frac{1}{1 + (\alpha h)^m} \right]^n, \quad m = 1 - \frac{1}{n}
$$

(1)

where a is the inverse of air-entry value, m and n are shape parameters, and Θ is the normalized volumetric moisture content, defined as

$$
\Theta = \frac{\theta_s - \theta_r}{\theta_s - \theta_r}
$$

(2)

where θs and θr are the residual and saturation volumetric contents, respectively. This constitutive relation allows to obtain the term $\beta = \partial \theta_s / \partial h$, which is a measure of the facility of water entering or leaving a soil element due to suction variations.

As a constitutive relation for the hydraulic conductivity, $k_s$, and the degree of saturation, $S_s$, it is used the power law model by Campbell (1974), expressed as

$$
k_s(S_s) = k_s^0 S_s^d
$$

(3)

where $d = 2b + 3$, b is a soil empirical exponent, and $k_s$ is the coefficient of saturated hydraulic conductivity.

The pore water pressure, $pwp$, field obtained from the flow analysis, and Bishop’s Method are used to calculate FS. It is noted that we assume a fixed failure surface, corresponding to the most probable surface for the stationary case. This surface represents the condition with the lowest FS, i.e., the most conservative condition.

3. Geometry, Initial Conditions and Soil Properties

The geometry of the analyzed slope is the hypothetical case presented by Otálvaro & Cordão-Neto (2013). The slope is divided into a mesh of 900 rectangular elements (approximately 1 m x 1 m). Slope inclination is 1.5 H: 1.0 V, width is 45 m, and height is 10 m. The water table level is that resulting from the stationary analysis, by imposing $pwp$ equal to zero at the slope foot, and a steady flow of 7 x 10⁻³ m/s entering on the left side. Figure 1 shows the water table position, and slope geometry.

Table 1 shows the values of the soil parameters used in the modelling. Mean, $\mu$, coefficient of variation, CV, and air-entry values are those estimated by Oliveira (2004) using the “Three-Sigma rule”.

The values of the soil water retention curve parameters are those by Oliveira (2004) and Otálvaro & Cordão-Neto (2013), shown in Table 2. The CV of n is determined

![Figure 1 - Geometry and initial conditions of the problem.](image)

**Figure 1 - Geometry and initial conditions of the problem.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>CV [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit weight $\gamma$ [kN/m³]</td>
<td>18.1</td>
<td>4.2</td>
</tr>
<tr>
<td>Effective cohesion $c'$ [kPa]</td>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>Effective friction angle $\phi$ ['°]</td>
<td>30</td>
<td>6</td>
</tr>
<tr>
<td>$k_s$ [m/s]</td>
<td>$1 \times 10^4$</td>
<td>90</td>
</tr>
<tr>
<td>$\theta_s^*$ [°]</td>
<td>30</td>
<td>15</td>
</tr>
</tbody>
</table>

*Angle indicating the shear strength change with matric suction change.
by Simota & Mayr (1996), and the CV of $\theta_1$ and $\theta_0$ are those found by Gitirana Jr. & Fredlund (2005). Table 3 shows the values of hydraulic conductivity curve parameters, where the mean of $b$ is obtained from Rueda (2008), and the CV from the information published by Clapp & Hornberger (1978). Figures 2a and 2b show the SWRC and the unsaturated hydraulic curve, respectively.

4. Sensitivity Analysis

In order to understand the general influence of mechanical and hydraulic parameters on $FS$, we perform a sensitivity analysis using FOSM. This method is used because it has a simple mathematical formulation and permits to quantify the influence of each variable independently, without requiring great computational efforts. Mechanical soil parameters considered as independent variables are: unit weight, $\gamma$, cohesion, $c$, friction angle, $\varphi$, and rate of increase in shear strength with suction, $\beta$. Hydraulic parameters are those defining the water retention and unsaturated hydraulic conductivity curves, i.e., $n$, $b$, $a$, $k_s$, and $r$.

![Figure 2 - Curves of (a) soil water retention, and (b) unsaturated hydraulic conductivity.](image)

Table 2 - Parameters of the water retention curve according to Van Genuchten model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\mu$</th>
<th>CV [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$ [-]</td>
<td>0.06</td>
<td>0.6</td>
</tr>
<tr>
<td>$n$ [-]</td>
<td>2</td>
<td>0.6</td>
</tr>
<tr>
<td>$a$ [-]</td>
<td>8</td>
<td>30</td>
</tr>
<tr>
<td>$\theta_1$ [-]</td>
<td>0.463</td>
<td>17</td>
</tr>
<tr>
<td>$\theta_0$ [-]</td>
<td>0.04</td>
<td>12</td>
</tr>
</tbody>
</table>

Figure 3 shows the percent variance of $FS$, $V[FS]$, of each parameter at 0, 8, 24 and 40 h, indicating that mechanical parameters have stronger influence on the variance at initial time, whereas hydraulic parameters become more important as time increases. For times greater than 24 h, $FS$ is dominated by hydraulic conductivity. The shape parameters of water retention and hydraulic conductivity curves have irrelevant influence when compared to $k_s$ and $\varphi$. It is noted that hydraulic parameters have no influence at time zero, because the mass matrix, representing the pore pressure variation with changes to the amount of water, is zero at that time.

5. Parameter Spatial Variation

According to the results obtained in the sensitivity analysis, the most influential mechanical parameters in $FS$ behavior are $c$ and $\varphi$, and the hydraulic parameters are $a$ and $k_s$, hence we focus on variations within these parameters. We assume that there is no correlation between any of the parameters.

To perform the variation, we assign random values of parameter to each node of the mesh. Then, the values of the four nodes composing each element are interpolated to obtain the variable value in each element. This interpolation is carried out with the same shape functions used to solve the flow equation at spatial dimension

$$P_{\text{elem}_m} = \sum_{i=1}^{4} p_i N_i(0,0)$$

where $P_{\text{elem}_m}$ is the parameter value in the element $m$, $p_i$ is the parameter value designated in node $i$ composing the element $m$, and $N_i$ is the shape functions of node $i$ evaluated in the local coordinate $(0,0)$. By using the shape function, a correlation between the parameter values attributed to the elements is maintained, avoiding abrupt changes among adjacent elements. Shape functions interpolate values for each one of the four nodes of the element, and each node belongs to several elements simultaneously. In other words, we performed a Kriging process. This process is
based on the basic idea that the value of a function at a given point can be calculated as a weighted average of the known values of the same function in the vicinity of the point (Oliver & Webster, 1999; Stein, 2012). In this case, the weights are given by the shape functions.

The node values for each parameter are assigned following normal and lognormal distributions, with mean and standard deviation shown in Table 2 and Table 3. In order to avoid negative or very small values leading to unrealistic values of $FS$, we truncate the normal distribution at zero for all the parameters except for $k_s$, which is truncated at $1 \times 10^{-9}$ m/s. The probability of negative values is spread within all the possible values.

Figure 4 shows the fields resulting from interpolation of the values of the friction angle, indicating that there is a correlation between the adjacent elements. Fig. 5 shows the correlograms exhibiting that an important correlation between elements is preserved to a distance of approximately 2 m. These results are similar for both normal and lognormal distributions, and using the Pearson and Spearman correlations.

Figure 6 summarizes the methodology used to vary spatially each parameter. After generating the parameter random field and keeping all other parameters and boundary conditions constant, we carried out a short-term flow analysis (a 24-h rainfall event). $FS$ is found with $pwp$ field and Bishop’s Method. Following the Monte Carlo Method, we perform the same procedure $n_e$ times for each parameter finding $n_i$ values of $FS$. $n_e$ is the number of necessary iterations required for Probability Density Functions, $pdfs$, of

![Figure 3 - Percent variance of $FS$ of each parameter.](image)

![Figure 4 - The field of value for friction angle assuming (a) a normal distribution, and (b) a lognormal distribution.](image)
FS to reach a constant shape. Each parameter is individually varied and then all simultaneously so that probability functions are determined for different times. By varying a single parameter, the minimum number of iterations is 1000, and by varying all parameters simultaneously, $n_i$ is 5000.

Figures 7 to 11 show the pdfs of FS by varying the parameters for different times. The horizontal axis is $\Delta FS$, i.e., the difference at each time between the generated FS (for each relative frequency) and the deterministic FS value. When $\Delta FS$ is zero, the FS of the pdf generated is equal to the deterministic FS. When $\Delta FS$ is negative, the FS value obtained is less than the deterministic FS; and when $\Delta FS$ is positive, the FS generated is greater than deterministic FS.

Figure 7 shows the pdfs of FS obtained by varying the cohesion parameter. The pdfs following both distributions have a mode similar to the deterministic FS, and a very low dispersion, indicating that variations of $c'$ are irrelevant for all the times analyzed. Although the cohesion value varies along the surface of rupture, when performing $n_i$ iterations, the values tend towards the mean.

Figure 8 shows the pdfs of FS when the friction angle is varied. The pdfs following both distributions have a mode similar to the deterministic FS, and their dispersion is greater than that obtained by varying $c'$. Similarly, the value of the friction angle tends towards the mean after many iterations, but as $\phi$ is multiplied by the effective stress from the stability model, which varies with time, the dispersion is greater.

Figure 9 shows the pdfs of FS by varying the air-entry value. When it is assumed a normal distribution, the pdfs of FS are similar to the ones obtained by varying $c'$, indicating a low relevance in the slope stability. However, when it is
assumed a lognormal distribution, the mode decreases as time increases.

The low dispersion of the \( pdf \)s of \( FS \) is due to the time when suction begins is controlled by the air-entry value, resulting in a null effect of \( a \) variation when the suction has greater values. The same happens in the soil below the water table level because it is saturated. Consequently, the influence of \( a \) is only important at points near the water table in the unsaturated zone.

Figure 8 - pdf of \( FS \) by spatially varying the friction angle following (a) normal distribution and (b) lognormal distribution.

Figure 9 - pdf of \( FS \) by spatially varying the air-entry value following (a) normal distribution and (b) lognormal distribution.

Figure 10 shows the influence of variations in saturated hydraulic conductivity. When \( k_s \) follows a normal distribution the mode and dispersion of \( pdfs \) of \( FS \) increase (\( \Delta FS > 0 \)) when time increases, decreasing the relative frequency of the mode.

When \( k_s \) follows a lognormal distribution, the dispersion also increases as time increases, but the mode decreases (\( \Delta FS < 0 \)), and the relative frequency of the mode increases. These differences are due to \( FS \) for the slope

Figure 10 - pdf of \( FS \) by spatially varying the saturated hydraulic conductivity following (a) normal distribution and (b) lognormal distribution.
studied being greater when hydraulic conductivity coefficient values are high and normal distributions assign higher probabilities to these values than lognormal distributions. The pdfs obtained by varying $k_s$ for both distributions show a strong influence on slope stability, since the dispersion is high and modes do not match the deterministic $FS$ values.

Figure 11 shows that by varying all parameters simultaneously, the pdfs of $FS$ have a similar behavior to the pdfs resulting from $k_s$ variation, confirming the strong influence of this parameter on slope stability. By assuming normal distributions for varying these parameters, dispersion is greater.

Figures 12 and 13 show the relative and cumulative frequencies and probabilities of failure, $P_f$, for each time assuming normal and lognormal distributions, respectively. Although the variability of air-entry value has little influence on stability, the pdfs of $FS$ generated by its variation move rapidly to the left, reaching high probabilities of failure (100% at 16 h) in a short time.

At 12 h, varying all the parameters simultaneously and following a normal distribution, the $P_f$ is less than the $P_f$ obtained by varying the air-entry value, cohesion coefficient, and friction angle, but it is greater than the $P_f$ obtained by varying $k_s$. This suggests that the $P_f$ decreases when hydraulic conductivity increases.
When varying simultaneously all the parameters following a lognormal distribution, the $P_f$ is less than $P_f$ obtained by varying $k_s, a$ and $c$, but it is greater than $P_f$ obtained by varying $\phi$. It suggests that spatial variability in the friction angle throughout the slope decreases failure probability. Varying the hydraulic conductivity and following a lognormal distribution, the pdf of $FS$ moves more rapidly to the left than when following normal distributions. This can be explained by higher probabilities being assigned to low coefficients by lognormal distributions.

As the conditions of the slope analyzed are very unfavorable, the failure probability, $T_p = 100\%$ for all cases at a specific time during the rainfall event. Table 4 shows the times when $P_f = 100\%$, indicating that the slope reaches this probability after 16 h when are varied $a$, $c$ and $\phi$, while when it is varied $k_s$ and all the parameters simultaneously, the time is greater and depends on the distribution assumed.

### Table 4 - Time required to reach a $P_f$ equal to 100%.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Normal pdf</th>
<th>Lognormal pdf</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_s$</td>
<td>20</td>
<td>16</td>
</tr>
<tr>
<td>$a$</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>$c$</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>$\phi$</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>$k_s, a, c$ and $\phi$</td>
<td>24</td>
<td>20</td>
</tr>
</tbody>
</table>

6. Conclusions

We used different approaches to analyze the influence of spatial parameter variability on slope stability. According to the sensitivity analysis based on FOSM the FS of the analyzed slope is most sensitive to friction angle and saturated hydraulic conductivity. The influence of mechanical parameters is greater for short times ($< 8$ h), and it decreases over time, increasing the influence of the hydraulic parameters. For times close to 24 h, the FS value depends almost entirely on the saturated hydraulic conductivity, because the location of the water table plays an important role in the analyzed slope stability, and it varies considerably with this parameter.

By assuming both normal and lognormal distributions to assign the values in each node, the influence of cohesion and air-entry values is irrelevant, while the influence of saturated hydraulic conductivity and all parameters simultaneously is very significant. The pdf of $FS$ obtained by varying all the parameters simultaneously is highly related to the $pdf$ obtained by varying $k_s$, showing the importance of conductivity in $FS$.

For the slope analyzed here, the higher the hydraulic conductivity coefficient, the greater the $FS$. This explains the contradictory pdf behavior of $FS$ by assuming normal and lognormal distributions. In the first case, the curves move to the right over time ($\Delta FS > 0$), indicating a lower probability of failure, while in the case of lognormal distributions, which are considered more realistic, curves move to the left ($\Delta FS < 0$) as time increases. This is because lognormal distribution gives higher probabilities to lower values of $k_s$ than normal distribution.

We note that these results are valid only for the hypothetical case analyzed, but the methodology can be easily implemented to evaluate the spatial variability of soil parameters when the finite elements method is used to solve the flow equation.

### Acknowledgments

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### References


**List of Symbols**

- $a$: inverse of air-entry value.
- $b$: hydraulic conductivity soil empirical exponent.
- $c$: soil effective cohesion.
- CV: coefficient of variation.
- FEM: Finite Element Method.
- FS: factor of safety.
- $h$: hydraulic head.
- $k$: coefficient of saturated hydraulic conductivity.
- $k_s$: coefficient of hydraulic conductivity.
- MSM: Monte-Carlo Simulation Method.
- $m$: shape parameter of Van-Genuchten equation.
- $n$: shape parameter of Van-Genuchten equation.
- $n_i$: number of iterations.
- $n_j$: shape functions.
- $P_{elem}$: parameter value evaluated in each element.
- $P_i$: parameter value in each node.
- $P_f$: probability of failure.
- $pwp$: pore water pressure.
- SWRC: Soil Water Retention Curve.
- $S_r$: degree of saturation.
- $t$: time.
- $VAR[.]$: variance.
- $\gamma$: soil unit weight.
- $\theta_r$: residual volumetric moisture content.
- $\theta_s$: saturated volumetric moisture content.
- $\Theta$: normalized volumetric moisture content.
- $\mu$: mean.
- $\phi$: friction angle.
- $\phi'$: rate of increase in shear strength with suction.