A Microstructural Cam Clay Model for Hydro-Mechanical Behavior of Unsaturated Soils


Abstract. In this paper, two new state variables are included in a conventional elastoplastic model, Modified Cam-clay, to incorporate the influence of pore size distribution changes in the mechanical behavior of unsaturated soils. The first state variable allows capturing the influence of large pore changes in the mechanical behavior, and is controlled by an evolution law that incorporates plastic volumetric strain. The second state variable is added to reproduce suction effects and the evolution of this variable is also controlled by plastic volumetric strain. The new approach is validated against a variety of experimental data of high porosity soils.

Keywords: high porosity soils, mechanical behavior, unsaturated soils, new state variables.

1. Introduction

Over the past decades, several constitutive models have been developed to reproduce the hydro-mechanical behavior of unsaturated soils. Alonso et al. (2010) state that nowadays, the development of constitutive models for unsaturated soils is tightly linked to the concept of effective stress. Indeed, the apparent success of this concept is also substantially connected to the fact that this effective stress takes into account the influence of the degree of saturation (Wheeler et al., 2003) or effective degree of saturation (Alonso et al., 2010).

On the other hand, some research trends aim at investigating the influence of the microstructure in the behavior of the soil; for instance: Prapahan et al. (1985), Griffiths & Joshi (1989), Delage et al. (1996), Qi et al. (1996), Penumadu & Dean (2000), Simms & Yanful (2005), Cuisinier & Laloui (2004), Kong et al. (2005), Li & Zhang (2009), Tarantino & de Col (2008), Delage (2010), Romero et al. (2011), Munoz-Castelblanco et al. (2012) among others. This paper aims to advance this discussion.

Figure 1 presents the results of a collapsible and porous soil of Brasilia, DF, Brazil (Silva, 2009). The figure illustrates some evidences of how loading and wetting can change the pore size density function (PSD). The PSD relates the log differential intrusion curve - obtained by mercury intrusion porosimetry - vs. entrance pore size D, which aids in the visual detection of the dominant pore modes. Results of three soil samples are presented: one as compacted on the dry side \( e = 1.16 \), a second sample loaded at constant water content up to 800 kPa of vertical stress \( e = 0.98 \), and the last one loaded at constant water content up to 800 kPa followed by collapse induced by saturation \( e = 0.77 \). It can be observed from Fig. 1 that both loading and collapse have an effect mostly in the largest pore sizes, which agrees with findings from Delage & Lefebvre (1984) and Griffiths & Joshi (1989). These authors reported that irreversible strains were caused by changes in the volume of the largest pores.

Based on the previous discussions, it can be concluded that any new constitutive model should take into account how the change in the fabric is affecting the mechanical response of the soil in terms of deformability and strength. This can be reached by introducing a new state variable associated with the soil fabric and the influence of large pore changes.

Figure 2 shows the evolution of the PSD when the soil is loaded and dried. It is clear that the PSD is differently affected by loading and suction changes. Moreover, the PSD of different soils is expected to have different responses.

Thus, this paper attempts to link the macroscopic behavior of the soil with its fabric by the introduction of two new state variables into a constitutive model. One of those variables is linked to the pore size distribution and the second one takes into account how the plastic volumetric strain affects the suction effect in the mechanical behavior. Those variables are used to adapt the Modified Cam-clay model to reproduce the behavior of unsaturated soils that present...
strong changes in their fabric, for instance, natural and compacted soils that display two distinct levels of dominants pores (large and small pores).

2. New Model Concepts

2.1. Analyses of the mechanical behavior of soils using PSD

As mentioned previously, PSD analyses may be used in the modelling process to help understand the hydro-mechanical behavior of soils and also assist in defining new state variables that provide a relationship between the macroscopic behavior and the microscopic response of the soil.

Figure 1 illustrates the case of a collapsible and highly porous soil, in which soil bonding effects are negligible. Thus, when analysing this graph it is possible to conclude that the region of the largest pores (macrofabric level) is easily destroyed when the soil is loaded and the smallest voids remain unchanged. A way to reproduce this hypothesis is to admit a superposition of effects between large pores and small pores, meaning:

\[ \delta e = \delta e^L + \delta e^S \]  

where \( \delta e \) is the total strain or the strain measured in the laboratory, \( \delta e^L \) is the strain that occurs due to changes of the large pore volume (macrofabric level) and \( \delta e^S \) is the strain due to changes of the small pore volume (microfabric level). Considering that initially \( \delta e^S \) is much greater than \( \delta e^L \), the total strain may then be associated only with \( \delta e^L \). However, as the macrofabric level is reduced, strain in the microfabric level will start to gain significance and for larger loading levels it will be predominant. Hence, it is possible to associate the mechanical response of the soil with the pore size distribution. In the following sections, the concepts necessary to complete the definition of the new model introduced in this paper are presented.

2.2. Classical admissible state region

In order to understand the new concepts presented in this paper it is necessary to introduce the idea of admissible states. Figure 3 shows two classical examples: the Mohr-Coulomb envelope (on the left) and the surface of elastoplastic constitutive models (on the right). In the latter, the surface delimits both the elastic region and the admissible states of the soil, as once the stress path reaches the surface it has to keep on it, given that beyond the surface is the region of impossible states. In this plot, \( e \) is the void ratio and \( p^\prime \) the mean effective stress.

In Fig. 3(b) a stress path can be seen where AB is elastic. If the condition of stress applied on the soil is such that it ends up in C' (an impossible state) the soil will be forced to rearrange its fabric leading it to C on the Normal Consolidated Line (NCL), which is the limit of admissible state in this space. This is a result of the inability of the soil fabric to sustain this stress state. Thereby, a rearrangement of the fabric occurs bringing the soil to a new state able to sustain the stress. This change is irreversible and it generates elastoplastic strains.

2.3. Intrinsic state

Some features of the soil such as anisotropy, suction, bonding and structural metastability modify the limit of admissible states of the soil, defined in the previous section. Although soils in their natural condition present some of those effects, it is possible to idealise a material with no alteration at all. This ideal state of the soil resembles the soil...
The ICL can be determined by two terms. The first one is the void ratio of the sample as if it were in the paste condition used to determine its liquid limit, i.e., $w(\%) = \lambda'$. This void ratio can be given by:

$$e \lambda' = G_s w_L$$

(2)

where $G_s$ is the specific gravity of the soil grains and $w_L$ is the liquid limit. Besides the void ratio, the other term required is its slope. In this paper it is assumed to be given by $\lambda'$. In section 3.1 a way to determine its value is presented.

### 2.4. Fabric effect

All features mentioned in the previous section change the domain of admissible states. As a result, the limit line for another condition incorporating one or more features does not correspond to the ICL. For all new features incorporated into the intrinsic state there will be a corresponding new limit line, which could reduce or increase the region of admissible states.

Figure 5 can help understand how the existing different pore levels could affect the soil behavior and the change in the admissible states. In the graph of Fig. 5, two samples of soils with an identical void ratio are shown, however with different pore size distributions. It is reasonable to as-
sume that the largest pores are more easily affected by a loading path than the smallest pores. Nevertheless, as the largest pores are being destroyed, both soils will tend to present similar behavior, as their pore size distribution curves will tend to be similar. This hypotheses was shown by Otálvaro (2013), Borges (2014) and Otálvaro et al. (2016).

Thus, the simplest fact that large pores exist reduces the area of admissible states, as shown in Fig. 4, which originates the new admissible line as a consequence of the fabric influence (FCL– Fabric Consolidated Line). The distance between the ICL and, the FCL is defined as the state variable $x'$. This new variable is used to measure the influence of the macrofabric (largest pores) level in the mechanical response of the soil and as soon as plastic strains start to develop, $x'$ will gradually begin to degrade. Therefore, the mechanical response approaches the behavior of the soil in the intrinsic state. State variable, $x'$, can be defined as:

$$x' = e' - e^0$$

(3)

where $e'$ is the initial void ratio of the sample in the natural condition and $e^0$ is the void ratio of the sample in the intrinsic state, defined by Eq. 2.

Figure 4 shows the path OABC that represents a loading in a soil sample in the intrinsic state, where only elastic strains occur in OAB, and elastoplastic strains occur in BC. For double structure soils the elastic region will be reduced, and elastoplastic strains will start occurring at A. At this point, large porosity effect is maximum and associated with $x'_0$. As shown in Fig. 4, when plastic strain occurs (path AD), $x'$ will gradually degrade according to its evolution law, given by:

$$x'_i = x'_0 e^{\left(-a'e\right)}$$

(4)

where $x'_i$ is the new state variable associated with macrofabric level strain; $x'_0$ is the initial value of $x'$; $a'$ controls the absolute rate of degradation; and $e_i$ is the plastic volumetric strain. The effect of parameter $a'$ can be seen in Fig. 6. High values of $a'$ increase the degradation effect of the macrofabric level in the mechanic response of the soil while $a' = 0$ brings the model to the conventional Modified Cam-Clay Model.

The plastic volumetric strains take place based on the ICL. Thus, it is necessary to project the current stress state point on the ICL and then plastic volumetric strain rates are given by:

$$de' = \frac{\lambda' - \kappa}{1 + e} dp_0'$$

(5)

where $de'$ is the plastic volumetric strain rate; $\lambda'$ is the slope of the ICL; $\kappa$ is the slope of the elastic path; $e'$ is the void ratio; $dp_0'$ is the pre-consolidation stress rate in the ICL; and $p_0'$ is the projection of the pre-consolidation stress on the ICL. Finally, pre-consolidation stresses are calculated by:

$$p_0' = p_0 e^{\left(-a'e\right)}$$

(6)

where $p_0'$ is the pre-consolidation stress on the FCL, and $x'$ measures the influence of the double structure on soil behavior. In Fig. 4, $p'_0$ and $p_0$ are the stress on the points A and B, respectively.

2.5. Suction effect

Suction effect shifts the limit of admissible states of the soil, amplifying its domain. This is due to the fact that suction effect increases soil capacity to sustain stresses without inducing plastic straining. As a result, the limit line for unsaturated soils (UCL) is shifted by $x'$ above the ICL which is a limit for fully saturated soils.

Figure 7 explains suction effect in the $p''$-$e$ space, where $p''$ represents the mean net stress. An unsaturated soil is loaded following elastic path DE, where point E is on the UCL. At this stage, $x'$ is the maximum and the region below UCL is elastic while above this line is a region of impossible states. From point E, elastoplastic strains start to occur on loading. Thus the UCL shifts towards the ICL and $x'$ will then be reduced originating path EF'. Therefore, the distance between UCL and ICL is reduced and consequently if a wetting path is applied to the soil, the resulting collapse will also be reduced. The pre-consolidated stress for both conditions, saturated and unsaturated, is defined by $p'_0$ and $p_0$ and these variables are used to define ICL and UCL.

State variable $x'$ will gradually degrade as plastic strains occur, in the same way as it occurs for $x'$, and following an equivalent evolution law given by:

$$x' = x'_0 e^{\left(-a'e\right)}$$

(7)
where $x^s$ is the suction effect on the soil; $x^s_0$ is its initial value; $\alpha^s$ controls the absolute rate of suction degradation; and $\varepsilon^p$ is the plastic volumetric strain. The effect of parameter $\alpha^s$ can be seen in Fig. 8.

Figure 7 shows two different stress paths followed by the same soil under saturated and unsaturated conditions. It can be observed that due to the expansion of the soil the initial void ratio of the saturated soil is greater than in the case of the unsaturated soil (the difference being equal to $\Delta$). Although both elastic stress paths have the same slope ($\kappa$), unsaturated soil starts yielding at a greater stress value, point E, compared to the stress value for the soil in the saturated condition, point B.

The physical meaning of suction effect degradation can be understood as a reduction of suction influence on compression in the global stiffness of the soil (on loading at constant suction, the degree of saturation increases and affects soil compressibility). This effect is in agreement with models that use Bishop’s stress as a state variable, such as Wheeler et al. (2003), Sheng et al. (2004), Alonso et al. (2010) and Della Vecchia et al. (2013).

From Fig. 7, it is also possible to obtain the relationship between saturated and unsaturated pre-consolidation stresses, i.e., the Loading Collapse curve (LC), as follows. The void ratio when mean net stress is $p$, on the ICL is given by:

$$N_i'(0) = \varepsilon_i + \Delta - \kappa' \ln \left( \frac{p^s_0}{p} \right)$$

where $N_i(0)$ is the void ratio when the mean net stress is $p$, on the ICL; $\varepsilon_i$ is the void ratio at point 1; $\Delta$ is the difference between the saturated and unsaturated initial void ratios for a suction unloading (wetting) in the elastic domain; $p^s_0$ is the pre-consolidation stress for the saturated path; and $p^s$ is a reference mean net stress, in this case 1 kPa. Similarly, the value of void ratio when the mean net stress is $p'$ on the UCL is given by:

$$N_i'(s) = \varepsilon_i + \Delta + \lambda' - \kappa' \ln \left( \frac{p^s_0}{p'} \right)$$

where $N_i(s)$ is the void ratio when the mean stress is $p$, on the UCL; $\kappa$ is the slope of the elastic path; and $p^s_0(s)$ is the pre-consolidation stress for the unsaturated path. Suction effect ($x^s$) is given by:

$$x^s = N_i'(s) - N_i'(0)$$

Thus, $x^s$ can be rewritten as:

$$x^s = -\Delta + \left( \lambda' - \kappa' \right) \ln \left( \frac{p^s_0(s)}{p^s_0(0)} \right)$$

and the Loading Collapse (LC) equation can be given by:

$$p^s_0(s) = p^s_0(0) e^{x^s(s)}$$

where $p^s_0(s)$ is the mean net stress on the UCL, and $p^s_0(0)$ is the mean net stress on the ICL. Similarly to what has been indicated for the state variable, $x'$, the authors consider that $x^s$ could also be described through analysis of the PSD curve. However, in the absence of more experimental data, $x^s$ is described as follows:

$$x^s = 1 - \left[ (1-r)e^{-\beta s} + r \right]$$

where $x^s$ is the suction effect degradation; $s$ is suction; $\beta$ and $r$ are parameters similar to BBM parameters (Alonso et al., 1990).

For moderately expansive soils, $\Delta$ can be calculated according to Alonso et al. (1990), as follows:
\[
\Delta = \kappa_s \ln \left( \frac{s + p_{am}}{p_{am}} \right) \tag{14}
\]

where \(p_{am}\) is the atmospheric pressure, \(s\) is suction, \(\kappa_s\) is the slope on the wetting path in the elastic region. The relationship presented in Eq. 14 is valid on the elastic region for wetting and drying paths only.

The analysis of Eq. 14 allows a better interpretation of \(\Delta\) and \(x'\) parameters, which control the mechanical response of the model under unsaturated conditions. While \(x'\) is affected by plastic volumetric strains (Eq. 7) and suction variations (Eq. 13), parameter \(\Delta\) is affected solely by suction variations (Eq. 14).

It is important to point out that \(\Delta\) can be described by any law that allows reproducing the volume changes that occur due to wetting and drying stress paths for expansive soils. In case these variations result in plastic strains, they will be taken into account through updates of \(x'\) and \(x\) state variables.

2.6. Fabric and suction coupling effects

In general, both effects mentioned previously appear combined in natural and compacted soils. As a result, limit states that were described separately above are now combined: the fabric (large and small pores) and suction effects. Path 0A is elastic and A is the yield point. If the soil sample is wetted, it follows path AB, indicating volumetric collapse. In this case, not only a reduction of suction effect is expected, but also destruction of the initial fabric of the soil. This results in the displacement of FCL towards the ICL. The new position of FCL can be calculated through the evaluation of plastic volumetric strains according to Eq. 4.

On the other hand, if the soil was loaded at constant suction from point A, the stress path would follow path AC, tending towards the UCL. This happens because the distance between UFCL and UCL reduces as plastic strains occur and \(x'\) degrades. However, besides degradation of the fabric effect, the suction effect is also degraded. Therefore, it could be observed that the path followed will depend on the values of \(\bar{d}'\) and \(d'_s\), which control the velocity of degradation effects of the fabric and suction, respectively.

2.7. Model formulation

The yield surface proposed for the new model is similar to the one presented by Alonso et al. (1990) to define the Barcelona Basic Model. It is described by the following equations:

\[
f^1(p, q, s, e^r) = q^2 - M^2(p^0_0(s) - p)(p + p^e) = 0 \tag{15}
\]

\[
f^2(p, q, s, e^r) = (s' - s) = 0 \tag{16}
\]

\[
p^0_0(s) = p^0_0(0)e^\left(\frac{\Delta + s'}{\kappa - \kappa'}\right) \tag{17}
\]

\[
p^e = \kappa^e s \tag{18}
\]

\[
x^r = 1 - \left[1 - (1 - r)e^{-\beta s} + r\right] \tag{19}
\]

where \(p, q\) and \(s\) are the stress state variables; \(e^r\) is the vector of plastic strain; \(s'\) is the suction increase yield locus; \(p^0_0(0)\) is the pre-consolidation stress for saturated condition soil; \(\lambda'\) and \(\kappa\) are the slopes of the Intrinsic Consolidation Line and Unloading Line; \(\kappa^e\) is the rate of cohesion increase with suction; and finally \(\beta\) and \(r\) are parameters related to suction effects on the LC yield locus (Eq. 17, which is equivalent to Eq. 12). In this work, the formulation presented will not take into account suction paths that go beyond \(s'\) (Eq. 16).

Figure 10 presents a 3D view of the yield surface under saturated conditions in the \(p^*\)-\(q\)-\(e\) space, in which it is possible to observe the effects induced by state variable \(x'\). In this figure, the Intrinsic Critical State Line (ICSL) and the Fabric Critical State Line (FCSL) are presented. Both lines tend to converge as \(x'\) is degraded, in the same way as it happens with ICL and FCL.

Hardening laws are defined relating plastic volumetric strain to the size of the yield surface. They can be expressed as follows:

\[
\frac{dp^i_0}{de^r} = \frac{1 + e^r}{\lambda' - \kappa} p^i_0 \tag{20}
\]
The new model considers associated flow rule, and then plastic strain can be determined by:

$$d\varepsilon_p = \Lambda \frac{\partial f^i}{\partial \sigma_i}$$

(25)

where $\Lambda$ is the plastic multiplier which is calculated as:

$$\Lambda = \frac{-a_i \frac{\partial a_i}{\partial Y} + c \frac{\partial s}{\partial Y}}{Y}$$

(26)

or

$$\Lambda = \frac{a_i D_{i,\lambda} \varepsilon_{i,\lambda}}{a_n D_{n,\lambda} b_n - Y} + \frac{(c - a_i D_{i,\lambda} H_{i,\lambda}) ds}{a_n D_{n,\lambda} b_n - Y}$$

(27)

where $a_i = \frac{\partial f^i}{\partial \sigma_i}$, $b_i = \frac{\partial g^i}{\partial \sigma_i}$ and $c = \frac{\partial f^f}{\partial s}$.

Terms $a_i$ and $b_i$ are easily determined, since they are similar to those terms of other constitutive models, such as BBM (Alonso et al., 1990), but here the associated flow, i.e., the $f^f = g^i$ is assumed. On the other hand, $c = \frac{\partial f^f}{\partial s}$ requires more attention, since its derivative depends on other terms:

$$c = \frac{\partial f^f}{\partial s} = \frac{\partial f^f}{\partial p_{i,0}(s)} \frac{\partial p_{i,0}(s)}{\partial \varepsilon_i} \frac{\partial \varepsilon_i}{\partial x^i}$$

(28)

The term $Y$ that appears in Eqs. 26 and 27, represents hardening and can be expressed as follows:

$$Y = \left( \frac{\partial g^i}{\partial \sigma_i} + \frac{\partial g^i}{\partial \sigma_j} \right) \frac{\partial f^f}{\partial p_{i,0}(s)} \frac{\partial p_{i,0}(s)}{\partial \varepsilon_i} \frac{\partial \varepsilon_i}{\partial x^i} + \frac{\partial f^f}{\partial p_{i,0}(s)} \frac{\partial p_{i,0}(s)}{\partial \varepsilon_i} \frac{\partial \varepsilon_i}{\partial x^i}$$

(29)

where all terms can be determined through Eqs. 6 and 13 to 23.

In the next section the model validation will be presented, which was accomplished through simulations of isotropic and oedometer stress paths. The model results were then compared to experimental test results.

### 3. Model Validation

To demonstrate the model capability, simulations results from different soils, where two of them were sampled at natural conditions, will be presented in the paper. In an initial stage, simulation results will be dealing with saturated soils. Then, unsaturated soil features will be addressed.

#### 3.1. Saturated soil modelling

Table 1 presents different soils and their main properties are indicated, as well as the parameters used for the saturated modelling. Some details on the choice of parameters is presented in the following.

The first variable to be discussed is the pre-consolidation stress. When analysing this variable, it should be remarked that most of the values presented in Table 1 are somewhat lower than the values used in the original studies (see references in the table). It is worth mentioning that in this paper, pre-consolidation stress is defined as the stress from which the soil no longer has linear-logarithm behavior with $\kappa$ slope, which differs from other criteria, as presented by Cui & Delage (1996), where pre-consolidation stress values are defined from the intersection of the extension lines with $\kappa$ and $\lambda$ slopes.

Parameter $\kappa$ is defined in the same way as most constitutive models, where it represents the elastic slope determined as:
where $\sigma_v$ and $\sigma_i$ vertical net stresses and $e_i$ and $e_o$ void ratios are defined in the elastic line under oedometer conditions. Parameter $\lambda'$ is defined as the slope of the elastoplastic (intrinsic) line, given by the following expression:

$$\lambda' = e_i - e_o \ln \frac{\sigma_0}{\sigma_i}$$

where $\sigma_v$ and $\sigma_i$ vertical net stresses and $e_i$ and $e_o$ void ratios are defined in the region where all the high porosity effect has been already destroyed, i.e. $x = 0$, which differentiates the new model from others. However, none of the test results used in this paper were defined to reach such a condition, therefore there is no clear evidence that the final portion of loading stage is in this condition. Figure 11(a) present $\lambda'$ values used in the simulations ($\lambda$ (model) in the figure) and the values obtained from the portion where it is believed $x'$ is close to 0, which corresponds to the highest vertical stresses of the tests ($\lambda$ (data) in the figure). In general, values used in the simulations are larger than experimentally determined values.

Variable $x'$ can be obtained from Eq. 3. Due to the high variability of liquid limit values ($w_l$) it is necessary to slightly adjust the values obtained from Eq. 3. Figure 11 (b) shows good agreement between values estimated from Eq. 3 ($x'$ (data) in the figure) and values used in the simulations to obtain best fit ($x'$ (model) in the figure).

On the other hand, parameter $d'$ controls the rate at which the high porosity effect is destroyed due to plastic volumetric straining. Eq. 4 is used to obtain $d'$, where at least two points of $x'$ are required. The initial value of $x'$ is known. Besides this, it is also possible to estimate its final value that corresponds to the difference between the void ratio of the sample and the void ratio on the ICL at high vertical net stresses. Plastic strains are also known, since an unloading path is usually carried out. Even using test data that were not specifically performed for the proposed model calibration, it was possible to obtain a good agreement as presented in Fig. 12. In this figure, the FCL and ICL lines of each soil considered are also plotted. The results presented in this work are a clearly better fit that the ones presented by Pereira (1996), Mascarenha (2008) and Silva (2009), the original sources of data, where Nonlinear Elastic models (Pereira, 1996) and Modify Cam-clay models (Mascarenha, 2008 and Silva, 2009) were used.

In the case of the Brasilia Clay (Fig. 12(d)), it is possible to note that $x'$ is nearly zero, which corresponds to a total destruction of the largest pores, as indicated in Fig. 1.

### 3.2. Unsaturated soil modelling

#### 3.2.1. Theoretical case

Although the hypotheses used in the new model are different from the ones used in BBM (Alonso et al., 1990),
the new model is able to reproduce, with some agreement, the results foreseen by BBM. Therefore, the first results presented in this section will be a comparison between the behavior foreseen by the new model and BBM.

Parameters used in BBM simulations are the same presented in Alonso et al. (1990) and they are summarised in Table 2 along with the parameters used in the new model. Parameters $\lambda'$ and $\kappa$ are assumed to display the same values as the ones proposed by Alonso et al. (1990). This means that $\lambda'$ can be considered the slope of ICL, since BBM is an extension of Modified Cam Clay. Variables ($p'_c(0)$ and $x'_u$) and $d_f$ parameter were determined to obtain best fit results along a saturated path.

Parameters $r$ and $\beta$ were obtained so that a best fit between LC curves of the new model and BBM were reached. Figure 13 shows a comparison between both LC curves. It is also possible to observe the high porosity effect in the curve, which corresponds to a drastic reduction of the pre-consolidation stress in the low suction range. In this case, it was compensated for by an increase of parameters $\beta$ and $r$ with respect to BBM values indicated in Table 2.

In Fig. 14 comparative results are presented for both saturated and unsaturated conditions, where the unsaturated paths considered were associated with an isotropic loading at constant suction $s = 200$ kPa, followed by saturation inducing volumetric collapse. The saturated path considered is a loading path starting from the same initial void ratio.
It is important to remark some aspects reproduced by the new model. First, the smooth transition that is observed between the elastic and intrinsic segments, which occurs during the degradation of the high porosity effect. A second point to be highlighted is the degradation of the suction effect (associated with the increase in degree of saturation as a consequence of loading at constant suction). This degradation results in a progressive approach between the saturated and unsaturated curves as plastic straining is induced. The velocity associated with this process is controlled by parameter \( a_s \) that also affects the collapsible response. This may be visualized through comparisons between Fig. 14(a) and (b), in which low suction degradation effects on compressibility (\( a_s = 1.5 \)) are compared to higher degradation effects (\( a_s = 10 \)). Finally, it is possible to observe that even with all other parameters kept constant, collapse value is significantly reduced when comparing both model results. In addition, it allows simulating a maximum collapse at intermediate vertical stresses (Fig. 14(b)).

### 3.2.2. Loading and wetting stress paths

In this section, loading and wetting results will be presented for compacted (Pereira, 1996) and natural (Mascarenha, 2008) soils. Table 3 presents the calibration of parameters obtained for Pereira (1996) and Mascarenha (2008) tests.

In general, the new model was able to adequately reproduce soil responses for different stress paths. Simulation results of Pereira (1996) data are presented in Fig. 15(a). It shows an oedometer path with application of vertical stress up to 400 kPa, followed by saturation and more application of vertical stress up to 800 kPa. The new model is able to reproduce precisely this path (an oedometer saturated path has already been presented in Fig. 12(a)).

Figure 15(b) presents the results of suction reduction paths under isotropic loading conditions at different mean net stresses. All samples started at the same initial suction (\( s = 370 \) kPa). The new model was able to satisfactorily reproduce collapse variation associated with different soil

### Table 2 - Parameters used in the new model.

<table>
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<tr>
<th></th>
<th>( \lambda' )</th>
<th>( \kappa )</th>
<th>( p_s^0 ) (kPa)</th>
<th>( p' ) (kPa)</th>
<th>( r )</th>
<th>( \beta ) (MPa)</th>
<th>( x_0^f )</th>
<th>( a' )</th>
<th>( a^r )</th>
<th>( \kappa^2 )</th>
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<td>BBM</td>
<td>0.2</td>
<td>0.02</td>
<td>200</td>
<td>100</td>
<td>0.75</td>
<td>12.5</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<td>160</td>
<td>-</td>
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<td>20</td>
<td>0.04</td>
<td>50</td>
<td>1.5</td>
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### Table 3 - Parameters calibrated for compacted and natural soils.

<table>
<thead>
<tr>
<th>Soil</th>
<th>( \lambda' )</th>
<th>( \kappa )</th>
<th>( p_s^0 ) (kPa)</th>
<th>( r )</th>
<th>( \beta ) (MPa)</th>
<th>( x_0^f )</th>
<th>( a' )</th>
<th>( a^r )</th>
<th>( \kappa^2 )</th>
</tr>
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<tbody>
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<td>Cataluña Silty Clay(^{(1)})</td>
<td>0.115</td>
<td>0.006</td>
<td>50</td>
<td>0.75</td>
<td>0.13</td>
<td>0.10</td>
<td>15</td>
<td>6</td>
<td>0.008</td>
</tr>
<tr>
<td>Residual silty sand derived from gneiss(^{(2)})</td>
<td>0.12</td>
<td>0.006</td>
<td>40</td>
<td>0.75</td>
<td>8.0</td>
<td>0.06</td>
<td>15</td>
<td>2</td>
<td>0.0</td>
</tr>
</tbody>
</table>

\(^{(1)}\)Mascarenha (2008), \(^{(2)}\)Pereira (1996).
stress states (different final suctions at different mean net stresses).

In Fig. 16 it is possible to observe the LC curve for the final condition of the wetting test at constant \( p = 200 \text{ kPa} \). In the same figure, the LC associated with BBM is shown (Farias & Cordão-Neto, 2010). There is a significant difference between both LC curves. This is due to the fact that while for BBM the LC curve depends exclusively on suction, in the new model the LC curve depends on suction and plastic volumetric strains which affect state variables \( x_f \) and \( x_S \). If state parameter \( x_S \) is strongly affected by plastic strains \((a_s = 10)\), then suction, as a consequence, will have less influence on soil behavior.

Two unsaturated stress paths reported by Mascarenha (2008) on a natural high-porosity soil were also used to perform simulations. In both tests, vertical net stress was increased up to 800 kPa (in these paths suction was kept constant, at 50 kPa and 400 kPa, respectively), followed by suction reduction until saturation. From this point on, tests followed the paths indicated in Fig. 17(a) and (b). The new model was able to adequately replicate soil behavior, including the volumetric collapse starting from different initial suctions (50 kPa and 400 kPa).

![Figure 15](image15.png)  
**Figure 15** - Pereira test simulations. (a) Loading and wetting paths (b) Wetting paths at different net mean stresses.

![Figure 16](image16.png)  
**Figure 16** - Wetting test simulation for constant mean net stress \( p = 200 \text{ kPa} \). Comparison between final LC curves, taking into account different \( a' \) values.

![Figure 17](image17.png)  
**Figure 17** - Mascarenha (2008) test simulations. Reproduction of collapse paths. (a) \( s = 50 \text{ kPa} \); (b) \( s = 400 \text{ kPa} \).
In Fig. 17 it is possible to observe that after saturation the soil is close to the ICL, which means $\chi$ is small. Figure 18 displays the evolution of the PSDs of the soil starting from natural conditions ($e = 0.71$) and after being loaded to 800 kPa at constant water content followed by wetting ($e = 0.53$).

As observed, there is a significant change in the pore size distribution, particularly in the region with pore diameters greater than 10,000 nm. Although the resulting PSD is still a bimodal curve, it is possible to observe that the loading tends to convert the bimodal curve into a unimodal one, which is the hypothesis assumed by the model in this paper. Loading and wetting induce an important reduction in the macropore volume, as well as shifting towards smaller values of the dominant macropore size. On the other hand, wetting induces some swelling of micropore volume (a new mode emerges in this micropore volume domain at a dominant pore size around 200 nm).

4. Conclusions

In this paper, a new constitutive model for unsaturated high-porosity soils is proposed, which incorporates two new state variables. These state variables have an important impact on the mechanical response of the soil and are associated with soil fabric (macropore volume that reduces on loading and wetting) and with suction. In order to introduce the model, fundamental aspects of intrinsic and admissible states of soils are first presented, together with pore size distribution changes along loading and wetting (collapse) paths.

The state variables are ruled by plastic volumetric strain. This allows capturing the influence on mechanical behavior of suction effects together with large pore changes (fabric effects) and degree of saturation changes (as a consequence of plastic straining). For example, during loading at constant suction the soil reaches a maximum collapse capability on wetting at an intermediate stress state. This is a consequence of the increase in stress (which increases collapse), and, on the other hand, to an opposite effect that increases degree of saturation and thus reduces the capability to collapse. Simultaneously tackling these effects (high porosity and suction) together with the evolution of plastic volumetric straining, enables a better reproduction of the behavioral features of high-porosity saturated and unsaturated soils.

The paper presented the model formulation together with the stress-strain relationships. Generally speaking, the model did not lose any of the predecessor’s characteristics (such as BBM, Alonso et al., 1990) and new ones were incorporated to enable better capacity to reproduce several aspects of soil behavior. Moreover, additional information associated with high porosity was incorporated in the Cam-clay model and BBM. This new information helps obtain some characteristics that are not captured in conventional models, such as smooth transition between pre- and normally consolidated locus.

As for the next steps, the authors consider that the model could be expanded, with little modifications, to expansive soils, and that state variables related to soil fabric could be directly obtained from the pore size distribution analysis. This way, it is expected that the macroscopic response of the soil could be better described through its microscopic behavior.

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References


Figure 18 - Evolution of pore size density functions (after Mascarenha, 2008).


**List of Symbols**

- **D:** entrance pore size
- **d:** elastic constitutive matrix
- **dp:** pre-consolidation stress rate in the ICL
- **d:** total strain or the strain measured in the laboratory
- **de:** strain that occurs due to changes of the large pore volume (macrofabric level)
- **de:** strain due to changes of the small pore volume (micromodel level)
- **de:** plastic volumetric strain rate
- **e:** void ratio
- **e:** void ratio of the sample in the intrinsic state
- **e:** initial void ratio of the sample in the natural condition
- **f:** yield surface (p and q axis)
- **f:** yield surface (p and s axis)
- **G:** specific gravity of the soil grains
- **h:** elastic constitutive vector
- **g:** plastic potential function
- **ICL:** Intrinsic Consolidated Line
- **LC:** Loading Collapse curve
- **M:** slope of critical state lines
- **N:** slope of the pre-consolidation stress on the UCL
- **N:** void ratio when the mean stress is 0 on the UCL
- **N:** void ratio when the mean net stress is 0 on the ICL
- **p:** net mean stress
- **p:** mean effective stress
- **p:** atmospheric pressure
- **p:** reference mean net stress
- **PI:** plasticity index
- **p:** projection of the pre-consolidation stress on the ICL
$p'(s)$: mean net stress on the UCL
$p'(0)$: mean net stress on the ICL
$p'_0$: pre-consolidation stress on the FCL
$p'_0(s)$: pre-consolidation stress for unsaturated condition soil
$p'_0(0)$: pre-consolidation stress for saturated condition soil
PSD: pore size density function
$q$: deviatoric stress
$r$: parameter defining the maximum soil stiffness
$s$: suction
$s'$: suction increase yield locus
$x'$: state variable associated with macrofabric level strain
$x'_0$: initial value of $x'$
$s^x$: suction effect degradation
$s'_0$: initial value of $s^x$
w: water content
$w'$: Liquid limit
$a'$: parameter that controls the absolute rate of degradation
$a$: parameter that controls the absolute rate of suction degradation
$\beta$: parameter controlling the rate of increase of soil stiffness with suction
$\Delta$: difference between the saturated and unsaturated initial void ratios for a suction unloading (wetting) in the elastic domain
$\varepsilon$: total strain or the strain measured in the laboratory
$\varepsilon'$: vector of plastic strain
$\varepsilon''$: plastic volumetric strain
$\phi$: friction angle
$\kappa$: slope of the elastic path
$k'$: rate of shear strength increase with suction
$\lambda'$: Slope of the ICL
$\sigma$: Net stresses
$\sigma'_0$: initial net stresses
$\sigma':$ vertical effective stresses