Kinematic Assessment of Multi-Face Round Slopes Using Hemispherical Projection Methods (HPM)

L. Jordá-Bordehore, R. Jordá-Bordehore, P.L. Romero-Crespo

Abstract. This paper presents a new approach to stability analysis of multi-face slopes, employing the concept of restricted daylight envelope. The methodology was applied to the back analysis of a real case of a curved, convex slope in metamorphic slates in Madrid, Spain. The Barton & Bandis (1990) criterion was employed for the field estimation of shear strength in rough discontinuities and to determine the friction cone, required for the kinematic analysis. The results suggest that the methodology reflects adequately the real situation and simplifies the studies of planar failure in complex geometries.

Keywords: rock mechanics, slope stability, stereographic projection, back analysis.

1. Introduction and Objectives

The existing methods for stereographic projection are widely extended for slope analysis in jointed rocks and possible failure modes (Wyllie & Mah, 2004; Hoek & Bray 1981; Hudson & Harrison, 1997; Gonzalez-Vallejo et al., 2002; Lisle, 2004; Lisle & Leysone, 2004; Koca et al.) since the first studies by Markland (1972) and John (1968, 1972). Most kinematic analyses of rock slopes consider exclusively single-face slopes (SFS), i.e., consider that the surface of the slope presents constant strike (Yoon et al., 2002). However, slopes can be found in nature with several faces or even curved faces, and it is not realistic to simplify these to “single-face” slopes.

In stereographic projection, the sliding envelope of a multi-face slope (MFS) is the union of single-face envelopes formed on the surface of the slope as proposed by Yoon et al. (2002). It is herein proposed to combine the methodology of Yoon et al. (2002) for multi-face slopes with conventional methodology for kinematic assessment of single-face slopes (daylight envelope and pole analysis) (Hudson and Harrison, 1997; Willie and Mah, 2004).

The work of Yoon et al. (2002) is one of the few existing methodological compilations on the kinematic assessment of MSF. These authors analysed the issue, both for planar and wedge failures, considering the orientation of the plane vector mode. The planar failure by definition applies to single planar discontinuities, in MSF and curved slopes only one planar surface is considered in the kinematic assessment. In the revised methodology proposed the assessment focus in the daylight envelope and pole plot of the discontinuities instead of the plane - vector analysis proposed by Yoon et al. (2002). In the case of wedges, the plane vector methodology is habitually employed; however, for planar failure, this approach is not common. In planar failures pole and daylight approach are more intuitive than plan vector or Dip Direction analysis. The work of Richards (2003) analyses wedges through the daylight envelope of poles. However, as indicated for the case of planar failure of slopes and toppling of strata, the most common approach is to analyse the location of discontinuity poles regarding the daylight envelope (Wyllie & Mah, 2004; Hudson & Harrison, 1997; Lisle, 2004).

The objective of this study is to present a methodology that unifies traditional criteria for rock slope analysis based on poles (Hudson & Harrison, 1997; Willie & Mah, 2004; Lisle, 2004) with the conclusions of the work of Yoon et al. (2002) for multi-face slopes. A study location was selected, which presented planar failure that could not be entirely justified by conventional single-face slope criteria. Since the slope has not a unique strike angle, we ask ourselves, which of them choose? In MFS, the slope can failed without any vertical release surfaces.

The suggested methodology for back analysis is effective to explain the mechanism of planar failure of real curved slopes, for slate rocks in El Atazar, Madrid.

2. Methodology

2.1. Kinematic assessment of rock slopes

The problems of wedge stability, planar failure, and toppling of strata are clearly three-dimensional-type problems. The visualization of the possible interactions between discontinuity planes and slopes is extremely complex to solve utilizing isometric projection, but it is considerably simplified with stereographic projection, where the planes are reduced to points (through poles). Kinematic assessment is based on the analysis of the relative orientation of...
the discontinuity planes or poles relative to the slope, establishing four main types of failure - planar, wedge, toppling or circular failure.

2.2. Kinematic assessment of planar failure

Five simple geometric criteria (Willie & Mah, 2004) must be fulfilled to guarantee the kinematic possibility of planar failure:  
1. The plane where the block slides must present a maximum strike difference, with respect to the face of the slope, of approximately ± 20°. This means the plane and the face of the slope must be noticeably parallel.  
2. Sub-vertical release surfaces commonly exist, which present negligible sliding resistance to define the lateral limits of the plane and of the block. When release joints occur planar failure could be considered as a peculiarity of wedge failures, with one set being the sliding plane and the second the release surface.  
3. The sliding plane must “daylight” on the slope face. This means that the plane must present lower dip than the slope.  
4. The dip of the sliding plane must be greater than the friction angle.  
5. The upper part of the sliding surface intersects the surface of the slope or finishes in a tension crack. Both tension cracks and release surfaces help to define the unstable block.  

It is practice to consider, in these simplified analyses, that the shear strength of discontinuities only relate to the friction component and that cohesion is negligible. The aforementioned conditions are depicted three-dimensionally in Fig. 1a, and a simple example is shown in Fig. 1b for stereographic projection utilizing equal angle projection.  

The friction cone has been added, which is built by counting φ degrees (friction angle of the plane) from the centre of the stereogram outwards. The poles located within the limit of the circle (cone) and the centre correspond to planes that present lower dips than the shear angle and therefore are kinematically stable. The possible sliding planes are laterally limited, as differences over 20° in direction or dip, for both sides, do not result in sliding (Willie & Mah, 2004) and may prevent sliding. The limits of the sliding plane boundaries in planar sliding failures are often ignored in stability analysis. The “final valid” envelope that encompasses the poles of the potentially unstable planes for planar failure is denominated restricted daylight envelope (Fig. 1b). The restricted envelope is the geometric site (focus) of all poles that represent a plane with dip direction located in the same semi space of the slope; also, these poles present lower dip than the plane, but still higher dip than the shear circle.  

The coloured zone in Fig. 1b corresponds to the geometric site of potential planar failure, including the dip condition, orientation regarding the slope, lateral limits and shear. All discontinuity poles that are located within this zone, referred to as restricted daylight envelope or restricted envelope with unstable poles, can yield planar failure.

2.3. Planar failure of a two-face slope

The sliding of a discontinuity plane in a two-face slope (Fig. 2a) occurs when the discontinuity daylights on any of the two faces, and meets the conditions given by Willie & Mah (2004). We consider a two face slope as relatively sharp corner or “nose like” where an abrupt change of strike occurs in a short distance. The corner is a “free face” for each of the faces and therefore for this point the release surface condition and the 20° difference in strike do not apply (Fig. 2a). Yoon et al. (2002) analysed the phenomenon via stereographic projection, but consider the maximum slope line of the sliding plane and the restricted envelopes of the plane by the friction cone. The criterion described in Hudson & Harrison (1997) - adding 20° to each side of the daylight envelope - was not considered by Yoon et al. (2002).  

The sliding mode shown in Fig. 2 is denominated by type 2 single plane sliding (Yoon et al., 2002). The sliding block is formed by the discontinuity or sliding plane and the two faces of the slope (RS and LS).

2.4. Multi-face slope - curved, convex slope

The same methodology applied to a two-face slope can be applied to MFS (Yoon et al., 2002). It is habitual to find curved slopes in highways (Fig. 3), both convex and concave (positive and negative MSF, respectively). The former are more habitual when rock structures are bypassed with minimum perturbation, while the latter are produced in highways with open slopes, in routes excavated using blasting techniques and circular shaped quarries. In these types of slopes, planar failure is produced if discontinuity daylights in any of the tangent directions to the slope (Fig. 4), and fulfils the condition that the discontinuity pole is located within the daylight envelope of the slope, i.e.:  
1. The sliding plane daylights in any tangent face of the slope, i.e., the plane presents lower dip than the slope.  
2. The dip of the sliding plane is higher than the friction angle.  
3. The superior part of the sliding plane intersects the upper face of the slope, or finishes in a tension crack.  
4. Maximum strike difference is 20° between the sliding plane and the tangent exterior faces of the slope.

It must be highlighted that some conditioning factors for simple slopes were eliminated, such as release surfaces and the 20° lateral limits for strike difference (which are only applied to the exterior faces). For the kinematic representation and analysis of a convex slope (nose-shaped), it is very useful to initially simplify the slope in three faces (Fig. 4). Facing the front of the slope, there is a left-tangent limit plane (LS) and a right-tangent limit plane (RS). Also,
a third plane situated between LS and RS can be considered, which characterizes the “roundness” or pointed shape of the slope (CS).

Figure 4b shows the analysis of a curved slope through stereographic projection - HPM in equal angle mode. Construction of the figure considered a simplification of the slope in three faces: left (LS), central (CS), and right (RS). The assumed friction angle - and therefore, friction cone - is common to all faces. Given that the interest lies in the analysis of planar failure through the poles, construction of the friction cone involved counting the angle $\phi$ from the centre of the stereogram outwards. Three daylight envelopes were built for each of the faces of the slope, between the centre of the stereogram and the pole of each one of the faces (in Fig. 4b: PRS, PCS and PLS). Twenty degrees (20°) are added to the exterior faces RS and LS due to slip limits, according to the single-face methodology. In this way, within the limits (±20°), the three daylight enve-
lopes, and the friction cone, the restricted daylight envelope of the multi-slope - round face is built, which will be the envelope for all poles that belong to potentially unstable planes. All the plane poles - strata or discontinuities - that fall within the interior of this differentiated zone in Fig. 3b are potentially unstable.

2.5. Field estimation of shear strength in rough discontinuities

The discontinuity friction angle is necessary to draw the friction circle - required in all kinematic analyses via stereographic projection (see Figs. 1b, 2b and 4b). Friction is obtained from the Barton & Bandis (1990) failure criterion, with \( \varphi \), linearized to obtain the instantaneous friction for the average stress state that acts in the potential sliding discontinuity. This failure criterion (Barton & Bandis, 1990) is a widely used empirical relationship for the modelling of shear strength in rock discontinuities. The criterion is based on two equations that represent the failure envelope. Equation 1 follows Barton & Choubey (1977):

\[
\tau = \sigma_n \tan \left( \varphi + JRC \log_{10} \left( \frac{JCS}{\sigma_n} \right) \right)
\]  

Barton & Choubey (1977) suggest that the residual friction of discontinuities \( \varphi_r \), can be obtained from Eq. 2:

\[
\varphi_r = (\varphi_b + 20) + 20 \left( \frac{r}{R} \right)
\]  

where \( \varphi_b \) is the basic friction angle of the failure surface. This value is tabulated and can be found in scientific literature (Barton, 1973; 1976, Barton & Choubey, 1977; Barton and Bandis, 1990). JRC is the joint roughness coefficient and JCS is the joint wall compressive strength (Barton, 1973 and 1976), and \( r \) is the Schmidt hammer rebound number on wet and weathered fracture surfaces and \( R \) is the rebound number on dry un-weathered sawn surfaces (Hoek, 2007).

**Field estimation of JRC - Joint Roughness Coefficient.** JRC is a number that can be estimated by comparing the appearance of a discontinuity surface with standard profiles published by Barton & Choubey (1977) and is widely reproduced in literature. The appearance of the discontinuity surface is compared visually with the profiles of the
standard figure for surfaces 10 cm long (Barton & Choubey, 1977) and the JRC value that most closely matches that of the discontinuity surface is chosen (Hoek, 2007).

The most utilized field technique for determination of joint wall compressive strength (JCS) uses the Schmidt hammer rebound equipment that was initially developed for concrete, but throughout 50 years has been systematically used for rocks, since the first works of Deere & Miller (1966).

Influence of scale on JRC and JCS: Barton & Bandis (1982) proposed scale corrections for JRC according to the following equation:

\[
JRC_x = JRC_0 \times \left( \frac{L_x}{L_0} \right)^{-0.02 \times JRC_0}
\]

where: \(JRC_0\) and \(L_0\) (length) refer to 10 cm laboratory samples or fragments, with the same longitude of the normalized profiles (Barton & Choubey, 1977), and \(JRC_x\) and \(L_x\) refer to the real size of the block and discontinuity analysed in situ.

Given that there is a great possibility of finding weaknesses in a large surface, JCS decreases when the scale is increased (Hoek, 2007). Barton & Bandis (1982) also propose an equation to correct the scale of JCS:

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**Figure 4** - (a) 3D scheme and plan view of a three-face slope and a round slope. For representation of the latter, it is habitual to utilize modified schemes based on Yoon et al. (2002) and build the slope with three faces. (b) Stereographic projection of a three-face slope. For the scheme, the following planes (dip direction/dip) were utilized: Left LS (225/60), Central CS (185/69) and Right RS (145/60), friction angle \(\varphi = 35^\circ\), using DIPSv5 software.
\[ JCS_n = JCS_0 \times \left( \frac{L_n}{L_0} \right)^{-0.03 + JRC_n} \]  

(4)

where: \( JCS_n \) and \( L_n \) refer to the real size of the block where the joint is located. Please note that the quotient must be coherent regarding the units, and therefore \( L_0 \) is considered in meters \( L_0 = 10 \text{ cm} = 0.1 \text{ m} \).

**Instantaneous cohesion and friction.** Given the historical development of rock mechanics, many analyses use the parameters of Mohr-Coulomb (cohesion \( c \) and friction angle \( \phi \)) to calculate safety factors regarding sliding:

\[ \tau = c + \sigma_n \times \tan \phi. \]  

(5)

The Barton & Bandis (1990) criterion (Eqs. 1 and 2) is a non-linear relationship and is not represented in terms of \( c \) or \( \phi \). Therefore it was necessary to develop a specific formulation that encompassed these aspects (Hoek et al., 1995); for example, kinematic analysis based on structural data of planes and wedges employs the concept of “friction cone” (Markland 1972; Hoek & Bray 1981; Hudson and Harrison, 1997; Willie & Mah, 2004; Lisle 2004) that required the value of the internal friction angle \( \phi \) of the discontinuity.

The instantaneous friction angle \( \phi_i \) for a normal stress \( \sigma_n \) can be calculated from the following relationship (Hoek, 2007):

\[ \phi_i = \arctan \left( \frac{\tau}{\sigma_n} \right) \]  

(6)

where:

\[ \frac{\partial \tau}{\partial \sigma_n} = \tan \left( JRC \log_{10} \frac{JCS}{\sigma_n} + \phi_i \right) - \frac{\pi JRC}{180 \ln 10} \left[ \tan^2 \left( JRC \log_{10} \frac{JCS}{\sigma_n} + \phi_i \right) + 1 \right]. \]  

(7)

Instantaneous cohesion \( c_i \) is calculated from

\[ c_i = \tau - \sigma_n \tan \phi_i. \]  

(8)

As Hoek (2007) remarked, the average normal stress \( \sigma_n \) that acts on the discontinuity planes is utilized to select the adequate values of \( c_i \) and \( \phi_i \) for a specific application.

**3. Study Site: Curved Slope with Planar Failure in Slates. El Atazar, Spain**

The El Atazar reservoir (Spain) has a double-curve vault dam, with 134 m height from foundation. It was one of the first of this kind to be built in Spain. It is located 100 km north of Madrid. Its construction started in

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**Figure 5** - The study zone is located in metamorphic terrain, schist, and black slates of the Medium Ordovician.
1965, with inauguration in 1972 by the dictator Francisco Franco. The final cost of the work was increased due to geotechnical issues with the Ordovician slates where it is set (Fig. 5). Currently a paved road crosses the dam and reaches the small village of El Atazar, which gives its name (Fig. 6).

The north side of the reservoir presented the most geotechnical problems throughout the construction; among other issues, this slope required important reinforcements of the rock mass, such as cables and walls. The highway bypasses a rocky prominence, due to a fold where planar failure was produced in a curved slope, at the time of construction. This sliding is nowadays completely stabilized (Fig. 6) and has been analyzed by the afore described methodology. In July 2014, a geomechanical station was carried out along the sliding (approximately 30 lineal meters, see Fig. 7) where besides censing the discontinuities (Fig. 8), field data were also collected on the main parameters for discontinuity resistance.

4. Results and Discussion

Three main join sets had been recognized in El Atazar curved slope (Table 1, Fig. 8).

Only in situ observations were utilized for the obtainment of the necessary parameters to establish shear strength in the discontinuities. Through the Schmidt hammer rebounds in altered joints, the value $r = 25.4$ was obtained, and for unaltered joints $R = 31.1$. The value of joint wall compressive strength $JCS$ was $R = 29.2$ (N-type hammer) and $JCS$ was obtained from the abacus of the employed equipment itself $JCS = 24$ MPa.

$JRC$ was estimated in the field from the normalized profiles and roughness tester or Barton’s comb (Fig. 9). Observationally, the roughness profile at macro scale of the main joint would be approximately 8-10. Equations 3 and 4 will be utilized for adequate consideration of the $JRC$ and $JCS$ scales. The height of the slope is 8 m but in the last 2 m there has not been any landslides, and therefore for the persistence of the joint $S_0$ the value of $L = 6$ m will be utilized.

For joint wall compressive strength $JCS$:

$$JCS = 24 \times \left( \frac{6}{01} \right)^{-0.03 - 0.5} = 384.$$  (9)

In the case scale effect is considered for $JRC_0$ determined by Barton’s comb, $JCR_0 = 4.35$ as demonstrated below. However, $JRC = 8 - 10$ will be employed in calculations, as determined in situ throughout the entire length of $S_0$ foliation.

$$JRC_0 = 15 \times \left( \frac{6}{01} \right)^{-0.02 - 0.5} = 435.$$  (10)

Determination of the residual friction angle through Equation 2 utilizes rebound values $r = 25.4$ and $R = 31.1$, plus the basic friction angle of the material: slate $\phi_b = 25 -}$
The average value within the interval is selected $\varphi_a = 27.5$. The values of residual friction and shear strength according to Equations 1 and 2 are:

$$\varphi_r = (27.5 - 20) + 20 \left( \frac{25.4}{31} \right) = 23.8 \sim 24^\circ \quad (11)$$

$$\tau = \sigma_n \times \tan \left[ 9 \times \log_{10} \left( \frac{384}{\sigma_n} + 24 \right) \right]. \quad (12)$$

Obtained of the equivalent Mohr Coulomb $c$ and $\varphi$ parameters involves linearization for a 6 m height and specific weight 0.026 MN/m$^3$ employing the Rocdata software (Rocscience). The result is instantaneous cohesion $c_i = 0.009$ MPa and instantaneous friction $\varphi_i = 33.85^\circ \sim 34^\circ$.

For the friction cone, $\varphi = 34^\circ$. Cohesion was not considered (Fig. 10).

The restricted daylight envelope (Figs. 10 and 11) was built for the curved slope of El Atazar, employing three auxiliary planes: central and two tangents (RS and LS). The stratification plane $S_0$ presents only one pole, which falls within the colored zone, corresponding to an unstable slope pole. Therefore, as expected, a slope that has slid is clearly unstable, given that, in addition, the pole is far from the friction circle - this implies a $FoS < 1$.

Figure 11 explains the procedure for the construction of the stereogram and kinematic assessment of Fig. 10: Fig. 11a, daylight envelope for a single face LS (Left side); Fig. 11b, expanded daylight envelope for three faces; Fig. 11c, daylight envelope restricted by a $34^\circ$ friction cone; Figure 11d, consideration of slip limits only for the outer tangential faces of the round-faced slope: LS-RS as internal slip limits are included into the single daylight envelopes. The pole of the bedding plane is located within the restricted daylight envelope, so it should be considered potentially unstable.

The methodology proposed is not opposed to the traditional solutions (e.g. Hoek & Bray, 1981; Lisle, 2004; Willie & Mah, 2004) but pretend to extend the traditional approach, which considers only single slopes, wherein instance release surface are required. In the study site of El

<table>
<thead>
<tr>
<th>Joint set</th>
<th>Dip direction (degrees)</th>
<th>Dip angle (degrees)</th>
<th>K Fisher distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>J1</td>
<td>55°</td>
<td>330°</td>
<td>103</td>
</tr>
<tr>
<td>S0 (Slate -Foliation)</td>
<td>48°</td>
<td>146°</td>
<td>114</td>
</tr>
<tr>
<td>J2</td>
<td>67°</td>
<td>108°</td>
<td>74</td>
</tr>
</tbody>
</table>

Figure 8 - Photography and simplified scheme of the main joint sets present in the El Atazar slope.

Figure 9 - Joint Roughness Coefficient in the foliation plane of the El Atazar slope: a) Barton & Choubey (1977) scheme, b) roughness $JRC$, according to a 10 cm profile, using “Barton’s comb” and macro roughness $JRC_m$. Both roughness profiles $JRC$ and $JRC_m$ ($JRC_{om}$) are super-imposed in a) to obtain the value of $JRC$.

Figure 10 - Restricted daylight envelope and estereographic analysis of the planar failure of the curved slope of El Atazar.
Atazar, the slope failed without any lateral - vertical - joint because the curved slope intersecting the planar failure surface generates a kinematic unstable block (together with the tension crack in the upper part of the slope).

5. Conclusions

Stereographic projection has been employed to represent and analyse the stability of multi-face rock slopes subjected to planar failure. The state-of-the-art has been reviewed and the existing methodology has been simplified through the combination of published methods for multi-face slope analysis (Yoon et al., 2002) and more recent simplified slope analysis models that consider daylight envelopes and poles (Koca et al., 2004; Richards, 2003). Back-analysis was carried out for a failed slope that presented a multi-face round convex slope. The applied methodology explained perfectly the situation produced.

As verified, it is habitual that real unstable slopes present complex geometries such as double-faces, multi-faces or even round-faces. These geometries can be analysed by Hemispherical Projection Methods (HPMs) and by the most usual and globally extended kinematic analysis that started with Markland (1972) and John (1968 and 1972).

The main contribution of the manuscript consists in reconsidering the methodology proposed by Yoon et al. (2002), which employs plane vector analysis for planar failure, and group it with pole vector analysis (widely utilized and well-known) through the utilization of daylight envelope for poles (Willie & Mah, 2004; Lisle, 2004).

In stereographic projections, all the conditions for planar failure are summarized in an envelope that is denominated “restricted daylight envelope”, in such a way that all pole planes that fall within the area yield potential planar failures.

It is difficult to give guidelines of when the curved slope approach applies - perhaps more case studies of failed slopes are needed.

It is considered that with a sharp turn of 20º the difference in strike criteria proposed by several authors and compiled by Willie & Mah (2004) cannot be applied for the inner part of the slope and failure surface. In these cases only the external 20º strike difference is considered (Figs. 10 and 11d).

Acknowledgments

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Figure 11 - Construction of the final restricted daylight envelope for a round-face slope using three auxiliary slope faces: Right, Central and Left (RS, CS, LS).
References


