On the Interpretation of the Bidirectional Static Load Test

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Abstract. The paper deals with the bidirectional test, aiming at interpreting its results, showing the factor that governs the upward movement and presenting approximate formulas to find the equivalent load-settlement curve of a conventional compressive test. The factor is the product \( c'\cdot k = (1 - c)\cdot k \), where \( c' \) and \( c \) are the Leonards-Lovell coefficients, related to the elastic shortening of the shaft under bidirectional and conventional static loading tests, respectively, and \( k \) is the relative stiffness of the pile-soil (shaft) system. The paper shows that when this product is constant the properly normalized upward curve is invariant, independently on shaft resistance distribution. Nomograms are presented to quickly determine the stiffness of the pile-soil (shaft) system. The paper shows that when this product is constant the properly normalized upward movement and presenting approximate formulas to find the equivalent curve of a conventional test.

1. Introduction

The Brazilian hydrodynamic expansive cell was developed by Silva (1983 and 1986) and since the 1980’s has been used in static load tests chiefly in bored piles. Its use spread worldwide after Osterberg (1989) and is known as “O-Cell Test” or the bidirectional test. One or more expansive interconnected cells are placed on the tip of a metal frame and introduced into the shaft, generally near the pile toe, and concreted together with the pile. The activation of the cells takes place hydraulically causing its expansion, pushing the shaft upward and the toe downward. The upward and downward movements can be measured at the level of the cells (bottom and top) with “tell tales” and at the top of the pile with dial gauges or displacement transducers.

The reaction system is provided by the pile shaft and the test is limited to the exhaustion of one of the pile capacity, tip or friction. Its execution is rapid and very high loads may be applied when associated with several cells.

The issue has been the subject of analysis by Alonso & Silva (2000), trying to simulate the equivalent conventional static load test. The usual procedure for establishing an equivalent conventional test curve consists in adding the shaft and toe applied loads which cause the same measured displacement, up and down. As the load causing the downward movement in the equivalent test induces greater elastic shortening than in the bidirectional test, additional movement must be added to the measured displacement in the bidirectional test, which constitutes the basis of the proposed approximate formulas to find the equivalent curve of the conventional test.

To validate these findings a mathematical model is used together with five case histories, comprising short to long piles. In one case a conventional compressive loading test was also available.

Keywords: bidirectional test, elastic shortening, equivalent curve, compressive loading test.

2. Shortening of Piles Under Compressive Loadings

The analysis will be initiated evaluating the shortening of piles subjected to both download conventional test and bidirectional test.

2.1. Shortening of piles during a download conventional test

To estimate the shortening of vertical piles, under axial compressive loading at the pile head \( (P_s) \), not necessarily at failure, the following expression may be used (see the list of symbols):

\[
\Delta e = \frac{Q_s}{K_r} + c \cdot \frac{A_t}{K_r}
\]

where \( Q_s \) and \( A_t \) are toe and shaft loads, respectively, so that:

\[
P_s = Q_s + A_t
\]
$K_r$ is the pile stiffness, with height $h$, cross sectional area $S$ and modulus of elasticity $E$, given by:

$$K_r = \frac{E \cdot S}{h}$$  \hspace{1cm} (3)

In Eq. 1 $c$ is the Leonards & Lovell (1979) coefficient, i.e., the ratio of the average value of the transferred lateral load (the hatched area of Fig. 1-a over pile height) and the total shaft load ($A_t$), i.e.:

$$c = \frac{A_t - \bar{A}_t}{A_t}$$  \hspace{1cm} (4)

The coefficient $c$ depends on the distribution of the unit shaft friction ($f_u$). If the shaft load is fully mobilized ($A_t = \bar{A}_t$), then $c = 0.5$ for $f_u = \text{const}$ along depth and $c = 2/3$ for $f_u$ increasing linearly with depth. Values of $c$ for other simple forms of distribution of $f_u$ can be obtained rapidly using the nomograms prepared by Leonards & Lovell (1979) or the equations proposed by Fellenius (1980), shown in Fig. 2. Note that $h_1$ and $h_2$ are the thicknesses of the softer and stiffer layers, respectively, and $f_{u1}$ and $f_{u2}$ are the corresponding shaft frictions.

### 2.2. Shortening of piles during a bidirectional test

For upward loads in the bidirectional test Eq. 1 changes to:

$$\Delta e = c', \frac{A_t}{K_r}$$  \hspace{1cm} (5)

where $c'$ is given by Eq. 4 but related to Fig. 1-b. A similar nomogram may be constructed for $c'$, as shown in Fig. 3 with the associated equations. Note that now $h_1$ and $h_2$ are

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**Figure 1** - Load distribution -a) Conventional test; and b) - Bidirectional test.

**Figure 2** - Nomograms for head-down loads (conventional loading test).
the thicknesses of the stiffer and softer layers, respectively, and \( f_{u1} \) and \( f_{u2} \) are the corresponding shaft frictions. Comparing Figs. 1-a and 1-b, on one hand, and Figs. 2 and 3, on the other hand, it can be seen that:

\[
c + c' = 1
\]

Note also that in general \( c' < c \), that is, the elastic compression of piles under O-Cell upward loads is smaller than the corresponding value for the download test. This is due to the fact that the O-Cell upward loads mobilize first the deeper and more resistant soils and later the upper softer layers.

3. Mathematical Simulation of Bidirectional Tests. The Upward Curve Invariance with \( c' \). \( k \)

3.1. The mathematical model

For the simulation of bidirectional tests a mathematical model developed by the author (Massad, 1995) for download conventional test is used. It is based on Modified Cambefort Relations (Fig. 4) and consideration is made of many aspects of load transfer phenomena, like the progressive failure, due to pile compressibility, and the residual stresses due to driving or subsequent loadings. As the piles in this paper are supposed to be cast in place and submitted to a single loading, residual stresses will be ignored.

A coefficient that measures the relative stiffness of the pile-soil (shaft) system was introduced and is defined as follows:

\[
k = \frac{A_u}{K_y \cdot y_1}
\]

where \( A_u \) is the ultimate shaft load; \( y_1 \), the pile displacement, of the order of a few millimeters, required to mobilize full shaft resistance. The coefficient \( k \) may be associated to the term \((\mu h)^3\) of Randolph & Wroth (1978). The model gave a further insight on pile behavior and led to a pile classification, with respect to the \( k \) values: “short” or rigid.

Figure 3 - Nomograms for upward loads (bidirectional test).

Figure 4 - Modified Cambefort Relations: a) shaft and b) toe.
(k ≤ 2); intermediate (2 ≤ k ≤ 8); and “long” or compressible (k ≥ 8).

3.2. Simulation of the Bidirectional test: invariance of the upward curves with $c^{'}.k$

To simulate the bidirectional test this model was changed to incorporate a soft upper layer ($f_{u2}$) over a deep more resistant soil ($f_{u1}$). For each layer along the shaft a relation as shown Fig. 4-a holds. Using subscripts 1 and 2 to distinguish them, it follows that:

$$f_{u1} \geq f_{u2} \quad \text{and} \quad A_{k1} = A_{k2} + A_{k2}$$

(8)

For both layers the value of $y_1$ (see Fig. 4-a) was assumed to be the same, so that the $k$ value (Eq. 7) may be associated to the whole subsoil along the pile shaft. Note that $K_r$ in Eq. 7 refers to the pile height above the O-Cell.

Ten cases of the bidirectional tests were simulated, varying the relations $f_{u2}/f_{u1}$ and $h_2/h_1$, as displayed on Fig. 5. For each case the soil below the O-Cell was the same, obeying the relation of Fig. 4-b, with the following parameters:

$$y_3 = y_1 \quad \text{besides} \quad \frac{R'.S_g}{K_r.k} = 0.80 \quad \text{and} \quad \frac{R'}{R} = 5$$

(9)

where $S_g$ is the cross sectional area of the pile toe.

Figures 6-a, 7-a and 8-a show, respectively, results for a rigid pile ($k = 1$ and Case 5), an intermediate pile ($k = 5$ and Case 3) and a long pile ($k = 10$ and Case 10). Note that the loads and movements are normalized with respect to $A_{k}$ and $y_1$, respectively. The range 0-3 corresponds to the initial pseudo elastic line of Fig. 4-a, with inclination $B$; the range 3-4 refers to the progressive mobilization of shaft resistance, from bottom to top, up to $y_t = y_1$ ($y_1$ is the pile top movement); point $M$ corresponds to the fully mobilization of $f_{u1}$ of the deep stiffer layer. For the unloading the analogous ranges are indicated through the points $3'$ and $M'$.

At the O-Cell level, the following relations hold for point 4:

$$P_{u4} = A_{k} \quad \text{and} \quad y_4 = y_1 + \frac{c^{'}.A_{j}}{K_r} = y_1 \cdot (1 + c^{'}.k)$$

then

$$\frac{y_4}{y_1} = 1 + c^{'}.k$$

(10)

Eqs. 10 still holds for cases of linearly increasing maximum unit skin friction with depth: in this case, $c^{'}, = 1/3$ (see Fig. 3-b).

The full mobilization of $f_{u1}$ of the deep stiffer layer initiates at points 3 of Figs. 6-a, 7-a and 8-a. The following relation holds:

$$\frac{y_3}{y_1} = 1$$

(11)

![Figure 5](image_url) - Analyzed cases.

![Figure 6](image_url) - Bidirectional test simulation for a rigid pile ($k = 1$, Case 5).
Moreover, the ratio \( P_{op}/Alr \) depends only on \( c'.k \), as shown in Figs. 9 and 10. Note that the case of linearly increasing maximum unit skin friction with depth (\( c' = 1/3 \)) is included in Fig. 9.

Taking also into account the last equation of Eq. 10, it follows that the normalized upward curves of the bidirectional tests are approximately invariant with respect to \( c'.k \), independently on the distribution of shaft resistances. This conclusion is confirmed by the plots of Figs. 11-a (\( c'.k \geq 0.5 \)), 12-a (\( c'.k \geq 2.3 \)) and 13-a (\( c'.k \geq 4.7 \)). Note that the Arabic numbers associated to the curves refer to the cases of Fig. 5. To stress this point, Fig. 14 was prepared showing different distribution of shaft resistances for cases 7 and 10 of Fig. 13, but with almost the same \( c'.k \), i.e., 4.9 and 4.8, respectively.

The conclusion about the invariance of the normalized upward movements related to \( c'.k \) is maintained even when one compares the cases of Fig. 15, with different values of \( k \), and also the cases of Fig. 16, whose skin friction distribution is presented in Fig. 17. Note that in the latter cases the product \( c'.k \) is close together.

In the attached Appendix more cases are presented showing that the mentioned invariance with respect do \( c'.k \) still holds, even for other transfer function besides the Cambefort Relation for the shaft friction.

It is worth mentioning that the movement at pile top (\( y_h \)) reaches the value \( y_1 \) at shaft failure load (\( A_s = A_{lr} \)), as can be seen in Figs. 6-a, 7-a and 8-a. Moreover, taking into account Eqs. 5 and 7, it follows:

\[
c' \cdot k = c' \cdot \frac{A_{lr}}{K_y \cdot y_1} = \frac{\Delta e_{max}}{y_1}
\]

that is, the ratio between the maximum elastic shortening of the pile shaft and the value of \( y_1 \) governs the upward curve of a bidirectional test.

4. Mathematical Simulation of the Equivalent Curve of the Conventional Test

The same mathematical model was also applied to simulate an equivalent download conventional test. Figs. 6-b, 7-b and 8-b show the equivalent curves for the cases of a rigid pile (\( k = 1 \)), an intermediate pile (\( k = 5 \)) and a...
long pile \((k = 10)\), respectively. The notable points 3, M and 4 (for loading) and 3' and M' (for unloading) are indicated in these plots. They have the same meaning as in the bidirectional tests, but with the progressive mobilization of shaft resistance going from top to bottom.

For point 4, the following relationships hold:

\[
y_4 = y_1 + \frac{c \cdot A_p}{K_y} + \frac{R' \cdot S_p \cdot y_1}{K_y}
\]

then

\[
\frac{y_4}{y_1} = 1 + c \cdot k + \frac{R' \cdot S_p}{K_y} \tag{13}
\]
From Eqs. 13 and 14 it follows that if \( k \) and \( c \) are constants, then \( c.k \) is also a constant and the coordinates of point 4 coincide, regardless the values of \( f_s/f_{s1} \) and \( h/h \) associated to \( c \) or to \( c' = 1 - c \) (Eq. 6). Additionally, as the soil below the O-Cell was supposed to be the same (Eqs. 9), the downward curves of the conventional tests are invariant with \( c'.k \), as shown in the plots of Figs. 11-b, 12-b and 13-b, taken separately.

Finally, as \( P_{st} \) is a function of \( k \) (Eq. 14), different values of \( k \) lead to distinct curves \( P_{st} - y_s \). This fact can be confirmed by comparing together Figs. 11-b, 12-b and 13-b.
5. Approximate Formulas to Determine the Equivalent Curve of the Conventional Test

To derive approximate formulas for the equivalent curve, considerations will be made using Fig. 18, supposing that the upward movement was measured at the top of the O-Cell and the load reached \( A_l \). Let \( P \) be a point of the upward curve with the coordinates \( y_f \) and \( A_r \).

Next, the elastic shortening of the shaft and \( y'_p \), an approximate measure of the pile head displacement, are computed as follows:

\[
\Delta e = \frac{c' \cdot A_i}{K_r} \quad (15)
\]

\[
y'_p = y_f - \Delta e \geq 0 \quad (16)
\]

Equation 16 is an approximate value of \( y'_p \) because \( c \) and \( c' \) depend on the amount of shaft friction mobilized during the loading. In a similar problem involving unloading in a conventional test, Massad (2001) showed that, for practical purposes, \( c \) may be approximated to the value corresponding to the maximum unit skin friction. It will be validated later on.

To simulate the download conventional test, \( y'_p \) is settled as the toe movement; it is associated to \( Q'_p \), as indicated in Fig. 18.

Finally, a pair \( y_o - P_o \) of the equivalent curve is determined by the equations:

\[
y_o = y'_p + \Delta e \cdot \frac{c}{c'} + \frac{Q'_r}{K_r} \quad (17)
\]

\[
P_o = A_i + Q'_p \quad (18)
\]

A first validation of Eqs. 17 and 18 is shown in Figs. 6-b, 7-b and 8-b: the curves of the approximate formulas and the mathematical model are in excellent agreement.

The usual procedure to construct the equivalent curve, also shown in Figs. 6-b, 7-b and 8-b, consists in adding the cell loads \((A_i + Q'_r)\) for equal measured movements up and down; no consideration is made to the correction of the measured movements due to elastic shortening of the pile.

The differences are greater for long piles as compared to short piles. As a matter of fact, for very rigid or very short piles, \( K_r \) is very large and both, \( \Delta e \) and \( Q'_r/K_r \) approach to zero and Eqs. 17 and 18 reduce to \( y_o = y'_p = y_f \) and \( P_o = A_i + Q'_p \); the usual procedure is valid.

If the movement of the pile head is available, instead of that of the top of the O-Cell, the procedure is analogous, as displayed in Fig. 19. And finally, if the movements at the pile head and on top of O-Cell are available, the measured \( \Delta e \) is used instead the value of Eq. (15).

It is worth highlighting that cases like those of Figs. 14 and 17 conduct to the same values of \( c \) and \( c' \). In fact, for the pair \( h/h = 0.1 \) and \( f_{s2}/f_{s1} = 0.7 \) (Case 7 Fig. 14-a) it follows from Fig. 3-a \( c' = 0.49 \) than \( c = 0.51 \), and for \( h/h = 0.9 \) and \( f_{s2}/f_{s1} = 0.7 \) (Case 10 Fig. 14-b), \( c' = 0.48 \) than \( c = 0.52 \). The same conclusion arises from Fig. 17: for Case 3 \( c' = 0.31 \) than \( c = 0.69 \) (see Fig. 3-a), very close to \( c' = 0.33 \) and \( c = 0.67 \) for the case of linearly increasing shaft resistance with depth (see Fig. 3-b). The conclusion is that different distribution of shaft resistance may lead to the same elastic shortening and so the same equivalent curve: the key factor is the elastic shortening of the shaft.

6. Practical Applications

Applications will be made to five case histories, comprising short to long piles, with heights and diameters varying from 11.5 to 41.0 m and 0.60 to 2.40 m, respectively. In one place a conventional compressive test was also available (see Table 1).

6.1. Continuous Flight Auger (CFA) piles in Belo Horizonte (Brazil)

Two CFA piles E 46 and E 46A, 0.60 m in diameter, were installed in Belo Horizonte, Brazil, 2.5 m apart. One of them was submitted to a conventional loading test and the other to a bidirectional test as shown in Table 1. The subsoil consisted of 1.8 m earth fill, on top of soft silty clay and clayey silt layers (SPT = 3 to 5), up to 10 m depth, and below a sandy silt residual soil (SPT = 17 to 30). The water table was at 8 m depth.
Figure 20 presents the results of the bidirectional test on E 46A pile, performed by Arcos. The O-Cell was placed at 14.0 m depth and the upward and downward movements were measured at pile head and at the base of the O-Cell, respectively.

Table 2 shows other relevant data. The value of $c$ was estimated using SPT data and the Décourt-Quaresma Method (1978) to determine the $f_u$ and the load distribution along depth, like shown in Fig. 1-b.

For the conventional loading test on Pile E 46, presented in Fig. 21, the Two Lines Method (Massad & Lazo, 1998 and Fonseca et al., 2007) was applied leading to an ultimate side friction ($A_r$) of 1,900 kN, $y_1$ equals to 0.35 mm and a toe stiffness of 3,000 kN/mm; some strain hardening was observed at the toe, probably due to the compression of a partially remolded soil, an outcome of pile installation.

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**Table 1** - Case Histories - general information of the piles.

<table>
<thead>
<tr>
<th>Type of test</th>
<th>Pile</th>
<th>Diameter (m)</th>
<th>Height (m)</th>
<th>Place</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional</td>
<td>CFA (E-46)</td>
<td>0.60</td>
<td>16.0</td>
<td>Belo Horizonte (BR)</td>
<td>Alonso &amp; Silva (2000)</td>
</tr>
<tr>
<td>Bidirectional</td>
<td>CFA (E-46A)</td>
<td>0.60</td>
<td>14.0 + 1.5 = 15.5</td>
<td>Belo Horizonte (BR)</td>
<td>Alonso &amp; Silva (2000)</td>
</tr>
<tr>
<td>Bidirectional</td>
<td>Omega (PC-02)</td>
<td>0.70</td>
<td>8.5 + 3.0 = 11.5</td>
<td>São Paulo (BR)</td>
<td>Fellenius (2014-a)</td>
</tr>
<tr>
<td>Bidirectional</td>
<td>Omega (PC-07)</td>
<td>0.70</td>
<td>7.2 + 4.3 = 11.5</td>
<td>São Paulo (BR)</td>
<td>Fellenius (2014-a)</td>
</tr>
<tr>
<td>Bidirectional</td>
<td>Bored (1)</td>
<td>0.90</td>
<td>16.0</td>
<td>Puerto Rico (USA)</td>
<td>Fellenius (2015)</td>
</tr>
<tr>
<td>Bidirectional</td>
<td>Bored (2)</td>
<td>1.25</td>
<td>40.0</td>
<td>Mississippi River (USA)</td>
<td>Fellenius (2014-b)</td>
</tr>
<tr>
<td>Bidirectional</td>
<td>Bored (3)</td>
<td>2.40</td>
<td>41.0</td>
<td>Tucson Arizona (USA)</td>
<td>Loadtest (2014)</td>
</tr>
</tbody>
</table>

**Table 2** - Results of the analysis.

<table>
<thead>
<tr>
<th>Type of test</th>
<th>Pile</th>
<th>$K_r$ (kN/mm)</th>
<th>$c$</th>
<th>$c'$</th>
<th>$k$</th>
<th>$c'.k$</th>
<th>Pile behavior</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional</td>
<td>CFA (E-46)</td>
<td>442</td>
<td>0.59</td>
<td>0.41</td>
<td>12.3</td>
<td>5.0</td>
<td>Long or Compressible</td>
</tr>
<tr>
<td>Bidirectional</td>
<td>CFA (E-46A)</td>
<td>505</td>
<td>0.57</td>
<td>0.43</td>
<td>7.6</td>
<td>3.3</td>
<td>Long or Compressible</td>
</tr>
<tr>
<td>Bidirectional</td>
<td>Omega (PC-02)</td>
<td>906</td>
<td>0.58</td>
<td>0.42</td>
<td>0.2</td>
<td>0.1</td>
<td>Short or rigid</td>
</tr>
<tr>
<td>Bidirectional</td>
<td>Omega (PC-07)</td>
<td>1069</td>
<td>0.58</td>
<td>0.42</td>
<td>0.2</td>
<td>0.1</td>
<td>Short or rigid</td>
</tr>
<tr>
<td>Bidirectional</td>
<td>Bored (Puerto Rico)</td>
<td>795</td>
<td>0.50</td>
<td>0.50</td>
<td>1.6</td>
<td>0.8</td>
<td>Short or rigid</td>
</tr>
<tr>
<td>Bidirectional</td>
<td>Bored (Mississippi)</td>
<td>614</td>
<td>var.</td>
<td>var.</td>
<td>0.9</td>
<td>var</td>
<td>Short or rigid</td>
</tr>
<tr>
<td>Bidirectional</td>
<td>Bored (Tucson)</td>
<td>4414</td>
<td>0.80</td>
<td>0.20</td>
<td>4.4</td>
<td>0.9</td>
<td>Intermediate</td>
</tr>
</tbody>
</table>

Legend: see appended list of symbols.
The toe did not reach failure. These values were confirmed by applying the Mathematical Model based on Cambefort Relations: the fitting between the measured and calculated curves is remarkable. The pile behaved as a long pile, with $k \sim 8$ (Table 2). It is worth pointing out that Décourt-Quaresma Method (1978) led to $A_r = 1,860$ kN.

As far as the Pile E 46A is concerned, submitted to the bidirectional test, the maximum side friction ($A_r$) up to 14 m was estimated to be 1,440 kN by the Décourt-Quaresma Method (1978), above the maximum upward load of 1,350 kN (see Fig. 20-a). Adjustments were made in the upward curve, as displayed in Fig. 20-a, eliminating the “jump” in the beginning and extrapolating at the end assuming a failure load of 1,440 kN. Table 3 was prepared using Eqs. 17 and 18 that led to the equivalent curve for the E 46A pile, as shown in Fig. 22. The segment of pile below 14 m, with a length of 1.5 m, was taken as a fictitious toe, with a transfer function given by Fig. 20-b. It includes the real toe and the side friction of the 1.5 m pile segment.

Next the same Mathematical Model (Cambefort) was applied to the bidirectional test on E 46A pile, using the ultimate side friction ($A_r$) of 1,440 kN, $y_1$ equals to 0.35 mm and toe stiffness of 150 kN/mm, this last figure gotten from the initial part of the “downward” curve (Fig. 20-b). The result is also shown in Fig. 22 together with the measured curve of the conventional test (Pile E 46). Again, the fitting is remarkable amongst the three curves up to the full mobilization of the shaft resistance in Pile E 46A. This is a second validation of the approximate formula, Eqs. 17 and 18. Figure 22 shows moreover that the fictitious toe resistance of Pile E 46A, given by Fig. 20-b, is much smaller than the toe resistance of Pile E 46, submitted to the conventional test, due to an unknown reason. This fact is supported by the result presented in Fig. 20-b. Finally, the application of the usual procedure as defined above led to unrealistic values of settlements.

### 6.2. Omega piles in São Paulo City (Brazil)

Fellenius (2014-a) presented the results (see Fig. 23) of bidirectional tests performed by Arcos at a site in São Paulo, Brazil, on two Omega Piles both with diameter 700 mm and embedment 11.5 m (see Table 1). Pile PC-02 was provided with a bidirectional cell at 8.5 m depth and Pile PC-07 at 7.2 m depth. The upward and downward movements were measured at pile head and at the base of the O-Cell, respectively. These and other information are presented in Table 2. The subsoil consisted of 2.5 m earth fill on top of layers of silty clay and sandy silt, SPT varying erratically from 5 to 15, and a very dense silty sand below 9 m depth. The water table was at 2 m depth. The value of $c' = 0.42$ was estimated using SPT data and the Décourt-Quaresma Method (1978) to determine the $f_r$ and the load distribution along depth.

The use of Eqs. 17 and 18 led to the results shown in Fig. 24. The agreement with the curves obtained by Fellenius with a software algorithm (UniPile) is remarkable. This is a third validation of the approximate formulas, Eqs. 17 and 18. Note that it was assumed fictitious toes below the O-Cells. As the pile is short or rigid, the usual procedure gave reasonable results.

### 6.3. Bored pile in Puerto Rico

Results of a bidirectional test on a 900 mm diameter bored pile, 16 m height (see Table 1), are shown in Fig. 25. The pile was drilled in clayey saprolite and socketed a short distance into weathered bedrock. The O-Cell was placed near the pile toe and the movements were taken at its base and top. The head down equivalent curves, obtained by Fellenius (2015) and with the application of Eqs. 17 and 18,
are again in quite a good agreement. This is a fourth validation of the approximate formulas. A value of $E = 20 \text{ GPa}$ was assumed for the pile and $c'$ was taken equals to 0.5 (Table 2).

6.4. Bored pile US82 Bridge across Mississipi River (USA)

A bidirectional test on a 1.25 m diameter 40 m deep bored pile (see Table 1) was performed at US82 Bridge across Mississippi River installed into dense sand. The O-Cell was placed near the pile toe and the movements were taken at its base and top. The results are shown in Fig. 26. Figure 27 displays the equivalent head-down curve for 3 hypotheses with respect to the parameter $c'$. Also shown is the curve obtained by the usual procedure. The differences are relatively small, because the pile behaved like a rigid or short pile: the value of $y_1$ was large and $k$ assumed a value close to 1 (see Table 2).
6.5. Bored pile - Tucson, Arizona (USA)

A bidirectional test was carried out in an instrumented bored pile, 2.4 m diameter and 41 m deep in Tucson, Arizona (USA) (See Tables 1 and 2). The O-Cell was placed near the pile toe and the movements were taken at its base and at pile head. Figure 28 shows the results. Based on the distribution of shear stresses given by the strain gages it was possible to estimate $c' = 0.20$. The value of $y_1$ was assumed to be 3.8 mm, $y_3 = y_1$, $R'_s / (K_r k) = 0.56$ and $R'/R = 10$.

The application of the Mathematical Model led to a good agreement with the measured up and downward load-movement curves (Figs. 28-a and b).

A good fitting (Fig. 29) was also obtained amongst the three equivalent curves: a) one computed with the Mathematical Model; b) the other with the approximate formulas (Eqs. 17 and 18), being this its fifth validation, and c) the curve referred by Loadtest Procedure (2001), mentioned before and shown in Loadtest (2014). The same cannot be said about the usual procedure: the $k$ value was relatively large, of the order of 4.4 (Table 2). Moreover, it is noticeable that: a) $c'k \approx 0.9$ and $P_s/A_{tr} \approx 0.74$, in agreement with the data of Fig. 9; b) $y_4/y_1 \approx 1.9$, in accordance with Eq. 10; and c) $y_3 = y_1$, as mentioned before.

7. Conclusions

The elastic compression of piles under O-Cell upward loads is generally smaller than the corresponding value for the download test or conventional loading test. Its estimation may be done using the coefficient $c' = 1 - c$, where $c$ is the Leonards-Lovell Coefficient for axial compressive loading applied at the pile head. Nomograms of $c'$ were presented for two patterns of shaft resistance distribution. The value of $c'$ may be estimated through the shaft resistance prediction by means of empirical methods based on SPT data. A reasonable estimation of the pile elasticity modulus ($E$) and its stiffness $K_r$ is also needed. Both parameter $c'$ and $K_r$ can be better determined by means of instrumentation.

The simulation of the bidirectional test using a mathematical model showed that the coefficient $c'$ plays an important role. The normalized upward curve is invariant...
when the product $c' \cdot k$ is constant, regardless may be the shaft resistance distribution and the separate value of $k$, the relative pile-soil (shaft) stiffness.

Approximate formulas to determine the equivalent curve were proposed, correcting the shaft elastic shortening induced by the upward loads in the bidirectional test. The measured or estimated displacement at pile top ($y')$ in the bidirectional test is set as the toe movement in the down-load conventional test. The load ($Q')$ at the base of the O-Cell associated to $y'$ is taken as the toe load. The shaft elastic shortening, measured or estimated in the bidirectional test, is corrected by the factor $c/c'$ and added to the pile compression due to $Q'$ and to $y'$ to get the pile head movement of the equivalent curve. The head load is obtained adding the O-Cell shaft load ($A_i$) to the O-Cell base load ($Q_p$), both values associated do $y'$. Application was made to five case histories, revealing the potentiality and easiness of the proposed procedure. It is shown that, when the pile is very rigid, the usual procedure gives good results, but, for compressible piles, the differences are relevant.

Appendix

The conclusion about the invariance with the term $c' \cdot k$ still holds even when one compares the cases of Fig. A-1, with different transfer functions, namely, Ratio Function (RF) and Cambefort (C) Relation, as displayed in Fig. A-2, and different resistance distribution along depth (Fig. A-3). In this figure RR and TR mean, respectively, rectangular-rectangular and triangular-rectangular shapes. The Ratio Function has been used by Fellenius (2014-a) to analyze the results of bidirectional tests and its general form is included in Fig. A-2. Note that:

a) while the Mathematical Model presented in this paper was applied to the cases related to the Cambefort Relation, the Coyle-Reese (1966) Method was used to deal with the Ratio Function; and

b) the value of $k$ associated to the Ratio Function cases was taken as:

$$k = \frac{A_i}{K_r \cdot (y'/2)}$$

in substitution of Eq. 7. The invariance of the upward curves at the top of the O-Cell ($y'$, of fig. A1) is remarkable.

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References


List of Symbols

\[ \begin{align*}
A & : \text{Total lateral (shaft) load} \\
A_r & : \text{Total lateral (shaft) load at failure} \\
B & : \text{Cambefort Parameters (see Fig. 4-a)} \\
c & : \text{Leonards & Lovell Coefficient (see Eq. 4)} \\
c' & : \text{Correlate of } c \text{ for bidirectional tests} \\
D & : \text{Diameter of solid pile} \\
E & : \text{Modulus of elasticity of the pile} \\
f & : \text{Unit skin friction} \\
f_1 & : \text{Maximum (ultimate) unit skin friction} \\
f_1';f_1'' & : \text{Layers 1 and 2} \\
f_2 & : \text{Residual unit skin friction} \\
h & : \text{Pile length embedded in soil} \\
h_1 & : \text{Thickness of the layers of the subsoil} \\
k & : \text{Relative stiffness of the pile-soil (shaft)} \\
K & : \text{Pile stiffness, as a structural piece} \\
P & : \text{Residual toe load} \\
P_{\text{reb}} & : \text{Residual toe load at the end of rebound} \\
P_s & : \text{Vertical load at the pile head} \\
P_{\text{reb}};P_{\text{rev}} & : \text{Associated to the initial shaft friction mobilization} \\
P_{\text{REV}};P_{\text{NCH}} & : \text{Associated to the full shaft friction mobilization} \\
P_{\text{max}} & : \text{Maximum value of } P_s \\
q & : \text{Toe pressure} \\
q_{\text{ult}} & : \text{Ultimate toe pressure} \\
Q & : \text{Tie load} \\
Q_{\text{max}} & : \text{Maximum value of } Q \\
R & : R';R_{\text{rev}}; \text{Soil stiffness at the pile toe (see Fig. 4-b)} \\
S & : \text{Cross sectional area of the pile shaft} \\
S_{\text{rev}} & : \text{Cross sectional area of the pile toe} \\
\text{SPT} & : \text{Standard Penetration Test blow count} \\
y & : \text{Pile movement} \\
y_1 & : \text{Movement of the pile at head and bottom} \\
y_2 & : \text{Movement of the pile at the top of O-Cell} \\
y_1; y_2 & : \text{See Figs. 4-a} \\
y_1'; y_2' & : \text{See Figs. 4-b} \\
\Delta \varepsilon & : \text{Elastic pile shortening} \\
\Delta \varepsilon_{\text{rev}} & : \text{Maximum elastic shortening of the pile shaft} \\
p & : \text{Elastic rebound measured at the pile head} \\
\theta & : \text{See Fig. A-2 of the Appendix} 
\end{align*} \]