Theoretical and Experimental Studies on the Resilience of Driven Piles

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Abstract. This paper deals with the resilient condition that may be reached by driven piles, during its installation as well as under static cyclic and monotonic loading tests. During the 1950’s Van Weele observed its effect in the context of a method for the separation of the toe and shaft loads at failure using the results of static cyclic loading tests on driven piles. Under this condition and based on a mathematical model that uses Cambefort’s Relations and takes into account pile compressibility and residual loads, this paper shows that there is a homothetic relation between the loading and the unloading-movement curves at pile head. A graphical construction is presented along with a numerical procedure allowing to improve the understanding of pile behavior and to determine significant parameters of pile-soil interaction under static or driving loadings. Application of the homothetic relations is made to several piles and particularly a new procedure was developed to analyze dynamic loading tests with a single blow record, considering the residual load at the toe.

Keywords: resilience, driven piles; homothety, residual loads, unloading.

1. Introduction

During pile driving or under static cyclic or monotonic loading a state of resilience may be reached in the soil along the shaft and at the toe, that is to say they respond elastically to the imposed loads. Without mentioning this term, Van Weele (1957) observed its effect in the context of a method for the separation of the toe and shaft loads at failure using the results of a static cyclic loading test on driven piles. The method requires: a) the full mobilization of lateral friction in the final cycles of loading-unloading; and b) the knowledge of the relationship between the mean and the total shaft load at failure. It consists of plotting a graph of maximum applied load as a function of elastic rebound, measured at the pile head. In the range associated to the shaft load full mobilization a straight line was obtained which enabled the mentioned separation. As will be dealt with later, the resilience effect was incorporated in the Smith Wave Equation Model (see, e.g., Rausche, 2002).

The application of Van Weele’s Method was extended by Massad (2001) to dynamic loading tests with increasing energy. In addition, a more general linear equation was derived relating the maximum applied load, shaft friction at failure, the residual toe load, the soil stiffness at the toe and the pile elastic rebound. This was done in the light of a mathematical model that incorporates, as load transfer functions, the modified Cambefort’s Relations and considers pile compressibility and the residual stresses.

In this paper it is shown how to use these findings and that the resilient state during repeated loading implies a homothety of the curves of loading and unloading movements at pile head.

2. The Resilient state According to Van Weele

The static cyclic loading test introduced by Van Weele (1957) consists of applying increasing loads in multiple cycles, such that at the end of each cycle the load is zero at the pile head. Figure 1 illustrates one complete cycle. In the load test analyzed by this author the concrete pile had a square section (0.38 cm x 0.38 cm), 14.05 m in length, and it was instrumented with strain gauges, installed in 4 levels along the shaft. In addition, movement measurements were taken at the pile head. There was also a portion of reference height, at the top, which allowed the assessment of the modulus of elasticity of the concrete, of the order of 38.3 GPa.

Figures 2 and 3 show: a) toe load \((Q_p)\) in function of toe quake \((C_3)\) and b) the maximum load in each cycle \((P_{\text{max}})\) as a function of the elastic rebound at the pile head \((\varepsilon)\) (see list of symbols at the end of the text). While Fig. 1 illustrates how \(C_3\) and \(\varepsilon\) were determined for one cycle, Fig. 2 allows the determination of the coefficient of subgrade reaction, as termed by Van Weele, or the Cambefort parameter \(R_{\text{sb}}\) that measure the resilient elastic response of the soil at pile toe (see Baguelin & Venon, 1971 and Massad, 1995, 2001). In general, and using the notations of Randolph & Wroth (1978), it is possible to write \(R = (2 \cdot D \cdot G_s) / [(1 - \nu) \cdot \eta \cdot S]\), where \(G_s\) is the Shear Modulus of the soil at pile toe (base).

The linear relationship displayed by Fig. 2 characterizes the effect of the resilience at the toe, which manifests itself in many geotechnical engineering problems with repeated loadings.
Based on Fig. 3 Van Weele proposed a method to separate the load in ultimate shaft friction and toe resistance. A condition for its application is the full mobilization of the shaft load in the last cycles. In the case of the load test analyzed by this author, this occurred above the maximum load ($P_{o, max}$) of 1000 kN, when the experimental points in Fig. 3 lined up. Van Weele obtained the following results:

- $A_r = 625$ kN for the ultimate shaft resistance, which compared with the measured figure of 608 kN;
- $Q_{p, max} = 1750 - 625 = 1125$ kN for the maximum toe load.

The Van Weele method still requires the knowledge of the relationship between the mean and the total shaft load at failure estimated in his case as 24.7% based on Begemann CPT.

3. The Meaning of the Van Weele’s Equation

Leonards & Lovell (1979), coincidentally aiming at taking advantage of the Van Weele’s achievement, proposed an equation to estimate the shortening of vertical piles, under axial compression loading at the head ($P_o$), not necessarily at failure, which can be written as follows:

$$\Delta e = \frac{Q_p}{K_r} + c \cdot \frac{A_t}{K_r}$$  \hspace{1cm} (1)

where $Q_p$ and $A_t$ are toe and shaft loads, respectively, so that:

$$P_o = Q_p + A_t$$  \hspace{1cm} (2)

$K_r$ is the pile stiffness, with height $h$, cross sectional area $S$ and modulus of elasticity $E$, given by:

$$K_r = \frac{E \cdot S}{h}$$  \hspace{1cm} (3)

In the expression 1 $c$ is the ratio of the average value of the transferred lateral load ($A_t - A_t'$) and the total shaft load ($A_t$), i.e.:

$$c = \frac{A_t - A_t'}{A_t}$$  \hspace{1cm} (4)

It depends on the distribution of the unit shaft friction ($f$). If the shaft load is fully mobilized ($A_t = A_t'$), then $c = 0.5$ for $f = \text{const}$ along depth and $c = 2/3$ for $f$ increasing linearly with depth. Values of $c$ for other simple forms of distribution of $f$ can be obtained rapidly using the nomograms prepared by Leonards & Lovell (1979) or the equations proposed by Fellenius (1980). For most common cases of heterogeneous layers $c$ varies in the range 0.5-0.8. In the case of Van Weele’s pile, $c = 1 - 24.7\% = 0.75$.

These equations remain valid for unloading, that is, in tension. Imagine that a static cyclic loading has reached the maximum load $P_{o, max}$ at the pile head and that, after unloading ($P_o = 0$), the elastic rebound, measured at the top, is $\rho$. In the usual form, it can be written:

$$\rho = C_2 + C_3$$  \hspace{1cm} (5)

where $C_2$ and $C_3$ are, respectively, the shaft and toe quakes (see Fig. 1).

Obviously, the absolute value of $C_2$, which is now elongation, is given by the expression 1 suitably rewritten, namely:
\[ C_2 = \frac{P_{\text{max}}^\text{A}}{K_y} + c \cdot \frac{A_y}{K_y} \]  \hspace{1cm} (6)

No consideration, for now, is being made with respect to any residual loads on the pile toe. The total shaft load decreases from \( A_y \) end of loading, to zero, end of unloading \( (P_y = 0) \). The same happens with the toe, whose reaction falls from \( Q_y = P_{\text{max}}^\text{A} - A_y \) to zero.

The value \( C \), may be expressed by the following equation:

\[ C_3 = \frac{P_{\text{max}}^\text{A}}{R_{\text{reb}} \cdot S_p} \]  \hspace{1cm} (7)

where \( S_p \) is the area of the pile toe and \( R_{\text{reb}} \) is one of the basic parameter of the second Cambefort Relation for unloading (“rebound”), as mentioned before. The hypothesis that \( R_{\text{reb}} \) is constant is plausible in a cyclic loading test as seen above in the context of Fig. 2.

Substituting Eqs. 6 and 7 into Eq. 5 and after several transformations results:

\[ P_y^\text{A} = A_y \left(1 - c \cdot \frac{d_{2R}}{K_y}\right) + d_{2R} \cdot \rho \]  \hspace{1cm} (8)

where \( d_{2R} \), given by:

\[ \frac{1}{d_{2R}} = \frac{1}{K_y} + \frac{1}{R_{\text{reb}} \cdot S_p} \]  \hspace{1cm} (9)

is a measure of the pile-soil (at the toe) stiffness.

If the shaft load in a cycle has been fully mobilized, \( i.e., A_y = A_y \) at the end of unloading \( (P_y = 0) \), the parameter \( c \) becomes constant, and the expression 8 turns into:

\[ P_y^\text{A} = A_y \left(1 - c \cdot \frac{d_{2R}}{K_y}\right) + d_{2R} \cdot \rho \]  \hspace{1cm} (10)

which is the meaning of the straight line of Fig. 3. In a plot of \( P_y^\text{A} \) vs. \( \rho \), there is a linear relationship, which occurred above 1000 kN in the case of concrete pile analyzed by Van Weele, illustrated in Fig. 3. Applying Eq. 10 in this case and taken into account that \( K_y = 394 \) kN/mm and \( c = 1-0.247 = 0.75 \) it follows that:

\[ d_{2R} = 148 \text{ kN/mm} \]  \hspace{1cm} (11)

\[ A_y = \frac{446}{l - (1-0.247) \cdot \frac{148}{394}} = 622 \text{ kN} \]  \hspace{1cm} (12)

and so:

\[ Q_{\text{max}} = 1750 - 622 = 1128 \text{ kN} \]  \hspace{1cm} (13)

which are in agreement with the figures gotten by Van Weele. Equation 9 leads to \( d_{2R} = 156 \) kN/mm, close to the value given by Eq. 11.

The existence of residual load has been known for a long time. Certainly it does not affect the pile load capacity but the skin friction and the end bearing values. According to Fellenius (2002), it is not easy to demonstrate its influence on test data and yet more difficult to quantify its effect. And this author adds: “Practice is, regrettably, to consider the residual load to be small and not significant to the analysis and to proceed with an evaluation based on “zeroing” all gages immediately before the start of the test. That is, the problem is solved by declaring it not to exist”.

In general, residual stresses can be dealt with a magnifier factor (\( \mu \)) (Massad, 1995), given by:

\[ \mu = 1 + \frac{P_y}{A_y} \]  \hspace{1cm} (14)

where \( P_y \) is the residual toe load, that is in equilibrium with the residual shaft friction, whose mean value is \( f'_{\text{reb}} \), and \( f'_{\text{a}} \) is the mean value of the ultimate unit shaft friction.

Note that \( \mu A_y = A_y + P_y \), \( i.e., \) the residual loads act as a shaft load due to the need to reverse the residual shaft friction. In general, this coefficient, that is greater than 1, is bounded by the smaller value between 2 and \( 1 + Q_{\text{max}}/A_y \), where \( Q_{\text{max}} \) is the toe load at failure.

Assuming now that at the end of a cycle of loading-unloading, with \( P_y = 0 \), a residual load \( (P_y^\text{reb}) \) arises at the toe of a driven pile, which is in equilibrium with the residual shaft friction, and introducing the factor \( \mu_{\text{reb}} = I + P_y^\text{reb}/A_y \), the expressions 6, 7 and 10 change to:

\[ C_2 = \frac{P_{\text{max}}^\text{A}}{K_y} \cdot \frac{\mu_{\text{reb}} \cdot A_y}{K_y} + c \cdot \frac{\mu_{\text{reb}} \cdot A_y}{K_y} \]  \hspace{1cm} (15)

\[ C_3 = \frac{P_{\text{max}}^\text{A}}{R_{\text{reb}} \cdot S_p} \cdot \frac{\mu_{\text{reb}} \cdot A_y}{K_y} \]  \hspace{1cm} (16)

\[ P_y^\text{max} = \mu_{\text{reb}} \cdot A_y \left(1 - c \cdot \frac{d_{2R}}{K_y}\right) + d_{2R} \cdot \rho \]  \hspace{1cm} (17)

Expression 17 is the general form of the Van Weele’s Equation (straight line of Fig. 3).

The difficulty in applying Eq. 17 is that \( c \) depends on the amount of shaft friction mobilized in the rebound. Massad (2001) presented results of a parametric study, using a mathematical model, to be presented later, based on the Cambefort’s Relations showing that at the end of rebound \( c \) varies from 0.4 to 0.5 (see Fig. 4-a) if the maximum unit skin friction \( (f') \) is constant; for friction full mobilization \( c = 0.5 \), as mentioned before. In Fig. 4-a:

\[ Q = \frac{C_3}{y_1} \]  \hspace{1cm} (18)
Thus, in a first approximation, \( c \) can be taken equal to 1/2, for \( f_u \) constant with depth.

If \( f_u \) varies linearly with depth, it can be proved that \( c \) ranges between 0.57 to 0.67, and again the last figure is associated with full mobilization of friction (see Fig. 4-b). Thus, in a first approximation, \( c \) can be taken equal 2/3 for \( f_u \) increasing linearly with depth.

4. Homothetic Relations Between the Loading and Unloading-Movement Curves at Pile Head

During pile driving or under static cyclic loading the state of resilience implies a homothetic relation between the loading and unloading ("rebound")-movement curves.

To demonstrate this similarity it will be used the already mentioned mathematical model that assumes the modified Cambefort’s Relations (Fig. 5-a and 5-b, with \( y_1 = \mu y \)) and takes into account pile compressibility (i.e. progressive failure) and residual loads due to driving or repeated loadings. It incorporates most of the features of the model developed by Baguelin & Venon (1971), in a simpler way and can be applied to bored, jacked or driven piles, first or subsequent loadings and unloadings. Initially, the soil will be admitted to be homogeneous, with \( f_u = \text{const.} \)

One advantage of using \( \mu \) is that it allows to take the residual loads as friction loads in the model (Massad, 1995).

4.1. Basic equations

A coefficient that measures the relative stiffness of the pile-soil (around the shaft) system was introduced by Massad (1995) and is defined as follows:

\[
k = \frac{A_{u}}{K \cdot y_1} \quad \text{and} \quad z = \sqrt{k} \tag{19}
\]

where \( y_1 \) is the pile displacement, of the order of some mm, required to mobilize the full shaft resistance (see Fig. 5-a). Note that: a) the maximum and the residual shaft frictions (\( f_u \) and \( f_{res} \)) are supposed to be constant along the pile; and b) the coefficient \( k \) is the term \((\mu h)^2\) of Randolph and Wroth (1978), with his notations, not to be confused with the symbol \( \mu \) used in this paper.

The model gave a further insight on pile behavior and led to a new pile classification, with respect to \( k \) values: “short” or rigid \((k \leq 2)\); intermediate \((2 \leq k \leq 8)\); and “long” or compressible \((k \geq 8)\) (see Massad, 1995).
The load ($P_o$)-movement ($y_c$) curve at pile head (see Fig. 6), during loading and unloading, may be expressed by
the equations shown in Table 1.

Reporting to Table 1 and Fig. 6, the range 0-3 corresponds to the initial pseudo elastic lines of Fig. 5, with inclinations $B$ and $R'$; the range 3-4 refers to the progressive mobilization of shaft resistance, from top to bottom, and also the point resistance up to $y_i = \mu y_c$; and finally the range 4-5 is the free development of toe resistance, $y > y_i = \mu y_c$. Point 5 is not necessarily associated with the failure load. The coefficients $\beta_1$ and $\beta_2$, depends on the characteristics of the soil-pile system and the toe parameter $R'$. But, for compressible piles (“long piles”) they approach 1 and the influence of the toe is very small in ranges 0-3 and 3-4. Note that if $\beta_2^\prime \equiv 1$, the range 3-4 turns to be a parabola. For very rigid piles, this range vanishes, that is, Points 3 and 4 coincide but the influence of $R'$ on range 0-3 is great. Note also that $\lambda'$ is the relative stiffness of the pile-soil (around the shaft and at the toe) system (Massad, 1995); and $d_i$ is the slope of the straight line 4-5.

For the unloading ranges 6-7, 7-8, and 8-9 (Fig. 6) the equations are similar in their form, but they differ: a) in the use of the appropriate Cambefort’s parameters for rebound, as shown on Figs. 5-a and 5-b; and b) if the loading stage ends further Point 4 (full mobilization of shaft friction), as assumed in Fig. 6, then $f_{re} = f_s$ and from Eq. 14 $\mu = 2$ at $P_o = P_{om}$ (see Massad, 1995).

4.2. Homothetic model for repeated loadings

Once resilient condition is reached, the following equations hold (see Fig. 5-b):

$$B = B_{reb} \quad \text{and} \quad R' = R_{reb}$$

Consequently, the loading and unloading curves tend to be homothetic in the ranges 0-4 and 6-8 of Fig. 6, a conclusion that comes from an inspection of the basic equations of Table 1. Ranges 4-5 and 8-9 are excluded, unless $R = R' = R_{reb}$, which does not occur necessarily.

An indication that the resilience is achieved can be seen by graphs like Fig. 7: there is a coincidence between the two curves at least in ranges 0-3 and 6-7.

4.2.1. Center and ratio of the homothety

An analysis of the formulae given in Table 1 shows that the ratio of homothety is $\mu/2$ and its center is in Point O,

$$\lambda' = \frac{\tanh(z) + \lambda'}{1 + \lambda'$}$

$$\beta_1 = \frac{\beta_1}{1 + \lambda' \cdot \tanh(z)} \quad \text{with} \quad \lambda' = \frac{R' \cdot S_p}{K_r \cdot \lambda'}$$

$$\beta_2 = \frac{\beta_2}{\tanh^{-1}(\beta_2)} = \left(\frac{P_o - 1}{P_o} \right) \cdot z - \tanh^{-1}(\lambda')$$

$$c = 0.5 \quad \text{and} \quad d_z = \frac{1}{1 + \frac{1}{R_S \cdot S_p}}$$

$$\beta_1 = \frac{\tanh(z) + \lambda_x}{1 + \lambda_x \cdot \tanh(z)} \quad \text{with} \quad \lambda_x = \frac{R_{reb} \cdot S_p}{K_r \cdot \lambda_x}$$

$$\beta_2 = \frac{\beta_2}{\tanh^{-1}(\beta_2)} = \left(\frac{P_{reb} - P_o}{P_o - 1} \right) \cdot z - \tanh^{-1}(\lambda_x)$$

$$c = 0.5 \quad \text{and} \quad d_{reb} = \frac{1}{1 + \frac{1}{R_{reb} \cdot S_p}}$$

Table 1 - Basic and auxiliary equations for the ranges of Fig. 6 (homogeneous soils).

<table>
<thead>
<tr>
<th>Range</th>
<th>Basic equation</th>
<th>Auxiliary equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-3</td>
<td>$P_o = \mu A_y \cdot \frac{y_c}{y_i}$</td>
<td>$\beta_1 = \frac{1}{\tanh(z) + \lambda'}$ with $\lambda' = \frac{R' \cdot S_p}{K_r \cdot \lambda'}$</td>
</tr>
<tr>
<td>3-4</td>
<td>$\frac{y_c}{y_i} = \left(1 - \beta_2^2 \right) + \frac{k}{2} \left(\frac{P_o}{\mu A_y}\right)^2$</td>
<td>$\beta_2 = \frac{1}{\tanh^{-1}(\beta_2)}$ with $\beta_2 = \frac{P_{reb} - P_o}{P_o - 1} \cdot z - \tanh^{-1}(\lambda_x)$</td>
</tr>
<tr>
<td>4-5</td>
<td>$\frac{P_o - \mu A_y}{\frac{(R' - R)S_p}{K_r \cdot \lambda_y}} = d_z$</td>
<td>$c = 0.5 \quad \text{and} \quad d_z = \frac{1}{\frac{1}{R_S \cdot S_p}}$</td>
</tr>
<tr>
<td>6-8</td>
<td>$P_{om} = P_o = 2A_y \cdot \frac{y_{om} - y_s}{2y_i}$</td>
<td>$\beta_1 = \frac{1}{\tanh(z) + \lambda_x}$ with $\lambda_x = \frac{R_{reb} \cdot S_p}{K_r \cdot \lambda_x}$</td>
</tr>
<tr>
<td>7-8</td>
<td>$\frac{y_{om} - y_s}{2y_i} = \left(1 - \beta_2^2 \right) + \frac{k}{2} \left(\frac{P_{om} - P_o}{2A_y}\right)^2$</td>
<td>$\beta_2 = \frac{1}{\tanh^{-1}(\beta_2)}$ with $\beta_2 = \frac{P_{reb} - P_o}{P_o - 1} \cdot z - \tanh^{-1}(\lambda_x)$</td>
</tr>
<tr>
<td>8-9</td>
<td>$\frac{(P_{om} - P_o) - 2A_y - R_{reb} \cdot S_p \cdot 2y_i}{\frac{(y_{om} - y_s) - c \cdot 2A_y}{K_r}} = d_{reb}$</td>
<td>$c = 0.5 \quad \text{and} \quad d_{reb} = \frac{1}{\frac{1}{R_{reb} \cdot S_p} + \frac{1}{K_r}}$</td>
</tr>
</tbody>
</table>

Note: The suffixes “$R'$” and “reb” refer to the rebound condition (see Fig. 5 and 6).
as displayed in Fig. 8. Note that \( \mu \) refers to \( P_s \) at the beginning of the loading stage (\( P_s = 0 \)).

For heterogeneous soils the auxiliary equations for ranges 0-3 and 6-7 are more complex but the basic equations are still valid; for the ranges 3-4 and 7-8, both equations, basic and auxiliaries are different. The equations of ranges 4-5 and 8-9 continue to be valid, with an appropriate \( c \) value, the parameter of Leonards and Lovell. In all of them the homothety is preserved.

Next the notable points of the homothety will be highlighted and its properties will be presented, using Fig. 8 as a reference.

4.2.2. Properties of the notable points

a) Consider the Point 4, associated to the full mobilization of lateral friction and toe reaction up to \( y = y_4 = \mu y_4 \). It has the coordinates:

\[
P_{4} = \mu A_{p} + R' S_{R_p} \mu y_{4} \tag{21}
\]

\[
y_{4} = \mu y_{4} + c \cdot \frac{\mu A_{p} + R' S_{R_p} \mu y_{4}}{K_{r}} \tag{22}
\]

b) Consider now Point 7, with the coordinates:

\[
P_{7} = \mu A_{p} \tag{23}
\]

\[
y_{7} = c \cdot \frac{\mu A_{p}}{K_{r}} \tag{24}
\]

It is located on the line through the origin and having a slope \( K_{r}/c \).

The line connecting Points 4 and 7 has a slope \( d_{gr} \) of Eq. 9, as shown below:

\[
\frac{P_{4} - P_{7}}{y_{4} - y_{7}} = \frac{(\mu A_{p} + R' S_{R_p} \mu y_{4}) - \mu A_{p}}{(\mu y_{4} + \frac{\mu A_{p} + R' S_{R_p} \mu y_{4}}{K_{r}}) - \mu A_{p}} \tag{25}
\]

\[
\frac{R' S_{R_p} \mu y_{4}}{\mu y_{4} + \frac{R' S_{R_p} \mu y_{4}}{K_{r}}} = \frac{1}{\frac{1}{1 - \frac{d_{gr}}{K_{r}}}} = d_{gr} R
\]

because \( R_{\text{reb}} = R' \), expression 20.

c) The Point \( P_{s} \), homothetic to \( P_{s} \), has coordinates:

\[
P_{s} = \mu A_{p} \tag{26}
\]

\[
y_{s} = \frac{2A_{p}}{K_{r}} \tag{27}
\]

It lies on the line passing through the point of maximum load \( (P_o^{\text{max}}, y_o^{\text{max}}) \) with a slope \( K_{r}/c \).

d) The Point 8, homothetic to Point 4, has coordinates:

\[
P_{8} = \mu A_{p} + R' S_{R_p} \cdot 2y_{1} \tag{28}
\]

\[
y_{8} = 2y_{1} + c \cdot \frac{2A_{p} + R' S_{R_p} \cdot 2y_{1}}{K_{r}} \tag{29}
\]

again because \( R_{\text{reb}} = R' \), expression 20. The line connecting the Points 7 and 8 has an inclination \( d_{gr} \), as can be proved similarly to expression 25. Moreover, this line intercepts the horizontal line passing through the point of maximum load \( (P_o^{\text{max}}, y_o^{\text{max}}) \) as indicated by the point \( P_{s} \) in Fig. 8. It is easy to prove that the abscissa of this point is given by:

\[
P_{s} = 2A_{p} \left( 1 - \frac{d_{gr}}{K_{r}} \right) \tag{30}
\]

e) The point \( P_{o} \), of ordinate \( y_o^{\text{max}} \) and belonging to the line passing through the point of maximum load \( (P_o^{\text{max}}, y_o^{\text{max}}) \) with a slope \( d_{gr} \), that is, parallel to the lines \( P_{4} \cdot 4 \) and \( P_{7} \cdot 8 \) (see Fig. 8), has an abscissa equals to \( P_{o} = P_{s} - \rho \cdot d_{gr} \) as it can be proved easily. This allows the estimation of \( \mu_{reb} \) at the end of unloading or rebound (\( P_s = 0 \)) using Eq. 17, i.e.:

\[
\mu_{reb} = \frac{P_{o}^{\text{max}} - \rho \cdot d_{gr}}{A_{p} \left( 1 - \frac{d_{gr}}{K_{r}} \right)} \tag{31-a}
\]
4.2.3. Another way to determine $\mu_{reb}$ at the end of unloading (rebound)

In the mentioned parametric study, Massad (2001) showed that for soils with $f_u = const$ along depth, i.e. $c = 0.5$, the following relations hold at the end of rebound ($P_o = 0$):

$$\mu_{reb} = 2 - \frac{2 - Q}{1 + \frac{2(1-c)k(r-l)}{2Q}} \leq r \text{ for } Q < 2$$

(31-b)

If $Q \geq 2$, $\mu_{reb} = 2$

where $Q$ is given by Eq. 18 and:

$$r = \frac{P_{max}}{A_p}$$

(31-c)

It can be proved that Eq. 31-b is also valid for $c = 2/3$ and it is postulated its soundness for any other $c$ value in the range 0.4 to 0.8. Figure 9 is a plot of Eq. 31-b.

5. Practical Applications

To illustrate the potentiality of the homothetic model, application will be made using experimental field data related to the driven piles presented in Table 2. The piles were arranged in groups, according to the type of test, i.e.: I) Static Cyclic Loading Test and Dynamic Loading Tests with Increasing Blow Energy; II) Static Loading Tests; and III) Dynamic Loading Tests with Single Blow Energy.

The application of the homothetic model assumes as initial known parameters:

a) the pile structural stiffness ($K$), given by Eq. 3;

b) the maximum applied load ($P_{max}$);

5.1. Piles of Group I

It is assumed that for the piles of Group I the Van Weele’s Equation (expression 17) is available, which means also that the shaft load was fully mobilized in the last cycles. It allows the determination of $\mu_{reb}A_p$ and $d_{sk}$ directly and of $R'.Sp/K$, using Eqs. 9 and 20. For the Van Weele pile, Fig. 3 and Eq. 17 with $c = 0.75$, estimated with the Begemann CPT, led to $d_{sk} = 148$ and $\mu_{reb}A_p = 621$ kN.

Next, it will be assumed that $\mu_{reb} = 2$, i.e., the value of $\mu$ is the same, for the beginning of loading ($P_o = 0$) and at the end of unloading (rebound) ($P_o = 0$). This hypothesis may be validated using Fig. 10, built up with the results of measurements with electric extensometers installed in the Amsterdam Pile by Van Weele (1957). Moreover, the results of the Dutch cone (CPT) fitted with a skin-friction jacket developed by Begemann revealed a total shaft load at failure ($A_p$) of 85 kN, which implies an average $f_u = 17.2$ kPa, a value very close to the one indicated in Fig. 10-a. The use of $\mu = 2$ in these conditions is not new: Massad (2001) and Fellenius (2001) did the same, in different ways, to estimate the true shaft resistance of a pile influenced by residual loads.

In this way, besides $P$, and its homothetic Point $P_{ho}$, the coordinates of Point 4 are also known (see Fig. 8): it is the interception of line 4-5 with line $P_A$, that has a known slope $d_{sk}$, as shown by Eq. 25. Using the homothetic relation, Point 8 is also known.

Applying Eqs. 15 and 16, with $c = 0.75$, the following values may be obtained: $C_1 = 3.41$ and $C_1' = 3.71$. Then $\rho = C_1 + C_1' = 7.11$ and $Q = C/(\mu_{reb}) = 2$. And finally, from Eq. 31-a results $\mu_{reb} = 2$, validating the assumed initial
Table 2 - General information of the piles.

<table>
<thead>
<tr>
<th>Group</th>
<th>Type of test</th>
<th>Local</th>
<th>Designation</th>
<th>Type of pile</th>
<th>Shaft soil (SPT)</th>
<th>$D_{1}$ or $L_{1}$</th>
<th>$D_{2}$</th>
<th>$h$</th>
<th>$K_{(\ast)}$</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Static Cyclic Loading Test</td>
<td>Amsterdam, Netherlands</td>
<td>VW</td>
<td>Static Cyclic Loading Test</td>
<td>Reinforced Concrete Solid Pile</td>
<td>Alternated layers of soft clay and sand, with peat</td>
<td>38x38</td>
<td>-</td>
<td>14.05</td>
<td>394</td>
</tr>
<tr>
<td></td>
<td></td>
<td>São Paulo City (Brooklyn)</td>
<td>BR-1</td>
<td>Static Cyclic Loading Test</td>
<td>Reinforced Concrete Pipe Pile</td>
<td>Porous Clay (3-4)</td>
<td>50</td>
<td>32</td>
<td>11.0</td>
<td>308</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>BR-2</td>
<td>Static Cyclic Loading Test</td>
<td>Reinforced Concrete Pipe Pile</td>
<td>Porous Clay (3-4)</td>
<td>60</td>
<td>40</td>
<td>11.0</td>
<td>373</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>BR-3</td>
<td>Static Cyclic Loading Test</td>
<td>Reinforced Concrete Pipe Pile</td>
<td>Porous Clay (3-4)</td>
<td>50</td>
<td>32</td>
<td>11.0</td>
<td>275</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>BR-4</td>
<td>Static Cyclic Loading Test</td>
<td>Reinforced Concrete Pipe Pile</td>
<td>Porous Clay (3-4)</td>
<td>50</td>
<td>32</td>
<td>11.0</td>
<td>275</td>
</tr>
<tr>
<td></td>
<td></td>
<td>São Paulo City (USP)</td>
<td>PRE-2 D</td>
<td>Static Cyclic Loading Test</td>
<td>Reinforced Concrete Solid Pile</td>
<td>Sandy Silt (Residual Soil)</td>
<td>50</td>
<td>32</td>
<td>8.7</td>
<td>333</td>
</tr>
<tr>
<td>II</td>
<td>Static Loading Tests</td>
<td>Santos Plain (Cosipa)</td>
<td>6</td>
<td>Static Loading Tests</td>
<td>Steel Pipe Pile</td>
<td>SFL (0-1)</td>
<td>35.6</td>
<td>33.7</td>
<td>31.5</td>
<td>69</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>9</td>
<td>Static Loading Tests</td>
<td>Steel Pipe Pile</td>
<td>SFL (0-1)</td>
<td>33.9</td>
<td>64</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>10</td>
<td>Static Loading Tests</td>
<td>Steel Pipe Pile</td>
<td>SFL (0-1)</td>
<td>36.0</td>
<td>83</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Santos Plain (Alamoa)</td>
<td>13</td>
<td>Static Loading Tests</td>
<td>Steel Pipe Pile filled with concrete</td>
<td>SFL and AT Clays (3-5)</td>
<td>46</td>
<td>-</td>
<td>45.0</td>
<td>134</td>
</tr>
<tr>
<td></td>
<td></td>
<td>São Paulo City (Penha)</td>
<td>P</td>
<td>Static Loading Tests</td>
<td>Steel Pipe Pile</td>
<td>Clay and Sand (17 to 19)</td>
<td>34.3</td>
<td>32.3</td>
<td>20.6</td>
<td>107</td>
</tr>
<tr>
<td></td>
<td></td>
<td>São Paulo City (USP)</td>
<td>PRE-2 S</td>
<td>Static Loading Tests</td>
<td>Reinforced Concrete Solid Pile</td>
<td>Sandy Silt (Residual Soil)</td>
<td>50</td>
<td>32</td>
<td>8.7</td>
<td>333</td>
</tr>
<tr>
<td>III</td>
<td>Dynamic Loading Tests with Single Blow</td>
<td>Santos Plain (Vicente de Carvalho)</td>
<td>K11</td>
<td>Dynamic Loading Tests with Single Blow</td>
<td>Reinforced Concrete Pipe Pile, coated with bitumen (23 m) (**)</td>
<td>Layers of clay with sand lenses (1-6), overlying sands with gravel (10-20)</td>
<td>80</td>
<td>50</td>
<td>34.6</td>
<td>223</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>O11</td>
<td>Dynamic Loading Tests with Single Blow</td>
<td>Reinforced Concrete Pipe Pile, coated with bitumen (30 m) (***)</td>
<td>Layers of clay with sand lenses (1-6), overlying sands with gravel and residual soil (10-20)</td>
<td>80</td>
<td>50</td>
<td>42.4</td>
<td>212</td>
</tr>
</tbody>
</table>

Notes: See attached list of symbols. (\*) For composite sections (for instance, concrete and steel), area-length weighted average is used. (**) 10 m Open Steel pipe at the toe, with section reducer ($\phi = 20$ cm). (***) 10 m Open Steel pipe at the toe.
value. The value of \( \rho \) may also be found using Van Welle Equation.

A graphical construction may also be done, as illustrated in Fig. 8. The values of the expressions \( 2A_{\rho,}(1-c.d_{\rho}/K) \) and \( P_{o, \text{max}} - \rho.d_{\rho} \) may be determined and so that of \( \mu_{\omega b} \) by means of Eq. (31-a). For the Amsterdam pile one can get \( \mu_{\omega b} = 446/[(1500-1054)/2] = 2 \), confirming again the assumption made at the start.

The results of these computations are summed up on Table 3 and presented in Figs. 11-a and 11-b. Although there is no coincidence between the two curves in Fig. 11-a, a close similarity exists between them. Figure 11-b shows that the homothety does occur, with the definition of its center (Point O).

The same procedure was used for the other piles of Group I. The Van Weele’s Equation could be set for all of them, as displayed on Fig. 12 and in Table 3, which allow a rigorous determination of \( \mu_{A_{\rho}} \) ans \( d_{\rho} \). In all of these cases the value of \( c \) was estimated using the SPT and its relation with \( f_{u} \) of the soil layers. Table 3 and Figs. 13 to 18 display

### Table 3 - Results of the analysis of Group I.

<table>
<thead>
<tr>
<th>Data</th>
<th>Parameter</th>
<th>Van Weele</th>
<th>201</th>
<th>BR-1</th>
<th>BR-2</th>
<th>BR-3</th>
<th>BR-4</th>
<th>Pre-2</th>
<th>Pile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input</td>
<td>( \mu ) (load)</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td></td>
<td>( K_r ) (kN/mm)</td>
<td>394</td>
<td>125</td>
<td>308</td>
<td>373</td>
<td>275</td>
<td>275</td>
<td>333</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( c )</td>
<td>0.75</td>
<td>0.50</td>
<td>0.55</td>
<td>0.55</td>
<td>0.55</td>
<td>0.55</td>
<td>0.50</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( P_{o, \text{max}} ) (kN)</td>
<td>1500</td>
<td>930</td>
<td>2080</td>
<td>2640</td>
<td>1850</td>
<td>1870</td>
<td>3470</td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>Eq. of line 4-5 ( P_o = d_y + d_{y_{o}} )</td>
<td>-</td>
<td>695+14y</td>
<td>693+105y</td>
<td>1186+74y</td>
<td>787+55y</td>
<td>1086+50y</td>
<td>2314+44y</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \rho ) (mm)</td>
<td>7.2</td>
<td>9.5</td>
<td>9.8</td>
<td>9.1</td>
<td>11.1</td>
<td>10.2</td>
<td>9.9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \mu_{A_{\rho}} ) (kN)</td>
<td>621</td>
<td>655</td>
<td>559</td>
<td>884</td>
<td>682</td>
<td>884</td>
<td>1617</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \mu_{\omega b} ) (mm)</td>
<td>3.70</td>
<td>2.01</td>
<td>1.55</td>
<td>1.05</td>
<td>1.63</td>
<td>2.60</td>
<td>1.01</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( k )</td>
<td>0.4</td>
<td>2.6</td>
<td>1.2</td>
<td>2.3</td>
<td>1.5</td>
<td>1.2</td>
<td>4.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( P_{o} ) (kN)</td>
<td>1482</td>
<td>773</td>
<td>1172</td>
<td>1478</td>
<td>1019</td>
<td>1398</td>
<td>2594</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( y_{o} ) (mm)</td>
<td>7.0</td>
<td>5.6</td>
<td>4.5</td>
<td>3.9</td>
<td>4.2</td>
<td>6.2</td>
<td>6.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( R.S_e ) (kN/mm)</td>
<td>237</td>
<td>15.8</td>
<td>160.2</td>
<td>92.3</td>
<td>68.8</td>
<td>61.1</td>
<td>50.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( R'.S_e ) (kN/mm)</td>
<td>237</td>
<td>59</td>
<td>395</td>
<td>567</td>
<td>207</td>
<td>198</td>
<td>972</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \mu_{\omega b} )</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( C_{\rho} ) (mm)</td>
<td>3.4</td>
<td>4.8</td>
<td>5.9</td>
<td>6.0</td>
<td>5.6</td>
<td>5.3</td>
<td>8.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( C_{y} ) (mm)</td>
<td>3.8</td>
<td>4.7</td>
<td>3.9</td>
<td>3.1</td>
<td>5.5</td>
<td>4.9</td>
<td>1.9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( Q )</td>
<td>2.0</td>
<td>4.7</td>
<td>4.9</td>
<td>5.9</td>
<td>6.8</td>
<td>3.8</td>
<td>3.8</td>
<td></td>
</tr>
</tbody>
</table>

**Figure 11** - Van Weele’s Pile: load-movement curves, loading up to 1500 kN and unloading.
the results obtained. Note that $\mu = \mu_{re} = 2$ for all these piles. The values of $R'$ and $R_{sp}$ were found using, respectively, the Eq. 9 and the equation of line 4-5 (see Tables 1 and 3).

Table 4 shows that the values of $f_i$ inferred by this analysis are consistent with those provided by the Brazilian Method of Décout-Quaresma (1978), based on SPT values. The closeness of the assessments is remarkable.

5.2. Piles of Group II

The piles of Group II were tested at some time after driving. The post driving residual forces may decrease in value due to creep and stress relaxation or to load history, although it is difficult to know to what extent. Rieke & Crowser (1987) suggested that 12 to 72 h are not sufficient to change the residual forces. It is worth mentioning, in this context, that the installation of additional piles may lift the...
piles already in position, decreasing their residual forces (Cooke et al., 1979).

For the Static Loading Tests of Group II, besides the initial parameters listed above, i.e. $K_r$ and $P_{\text{max}}$, it is supposed to know:

a) the elastic rebound ($\rho$) at pile head; and

b) the coordinates of Point 4 ($P_o, y_o$);

c) the equation of the straight line 4-5, i.e.,

\[ P_o = d_1 + d_2 y_o, \]

and

d) the parameter $d_{\text{ref}}$.

To help determining Points 4 and 5 a plot of $y_o$ as a function of $(P_o)^2$ may be drawn, as illustrated in Fig. 19 for 2 piles of Group II. For long piles, this relation turns to be a straight line in the range 3-4, because, as observed above, the curve $P_o - y_o$ approaches a parabola.

As a consequence, Point $P_o$ may be set as the intersection of the line passing through Point 4 with a slope $d_{\text{ref}}$ and the line passing through the origin with a slope $K_r/c$ (Fig. 8).

Most of the piles of Group II were installed in Santos Plain (Table 2), comprising two different soil layers, fluviomarine SFL clay overlying sandy silts (residual soils form Gneiss). For this soil profile, local experience reveals a value of $c$ around 0.65. Using the Leonards and Lovell’s Nomograms or Fellenius Equation the values of the ratio between $f_o$ of the first layer and $f_o$ of the second layer were found, as indicated on Table 5. The case of Cosipa 10 was an exception, because the residual soil layer was missing: the toe of the pile was in contact with the rock and a value of $c = 0.50$ was taken. Finally, for the other 2 piles from São Paulo City (see Table 2) the soil was supposed to be homogeneous, with $c = 0.5$.

The calculation is iterative in $\mu$ (in the beginning of loading). Therefore, a value of $\mu$ between 1 and 2 is adopted initially.

In this way, the following parameters may be determined, in sequence:

i) $A_o = \mu A_o / \mu$ because $\mu A_o$ is the abscissa of Point $P_o$ (Eq. 23);

ii) soil toe stiffness $R.S_p$, computed by the equation

\[ R.S_p = (1/d_2 - 1/K_r)^{-1}; \]

iii) $R'.S_p = R_{\text{ref}} S_p$ using the known value of $d_{\text{ref}}$ and Eq. 9;

iv) $\mu_{\text{ref}} A_o$ using expression 31-a;

v) $Q$ determined iteratively by means of Eqs. 15, 16, 18 and either Eq. 31-a or Fig. 9 and then the values of $\mu_{\text{ref}}, C_2$ and $C_3$; and

vi) $A_o = \mu_{\text{ref}} A_o / \mu_{\text{ref}}$.

Step i is compared with step vi. If the resulted $A_o$ values are different, another interaction in $\mu$ is done until convergence is achieved.

Table 4 - Comparison between $f_o$ assessments.

<table>
<thead>
<tr>
<th>Piles of group I</th>
<th>SPT average (Shaft)</th>
<th>$f_o$ (Décourt-Quaresma)</th>
<th>$f_o$ (Present analysis)</th>
</tr>
</thead>
<tbody>
<tr>
<td>201</td>
<td>10.0</td>
<td>43</td>
<td>54</td>
</tr>
<tr>
<td>BR-1 to BR-4</td>
<td>3.8</td>
<td>23</td>
<td>20</td>
</tr>
<tr>
<td>PRE-2</td>
<td>16.0</td>
<td>63</td>
<td>59</td>
</tr>
</tbody>
</table>

Figure 18 - Pile PRE 2 D – Group I.

Figure 19 - Illustrations on the determination of point 4.
Reporting to Fig. 8, a graphical solution is presented for the piles of Group II. It consists of the following steps:

i) plotting Point $P_s$ as the intersection of the line passing through Point 4 with a slope $d_2$, and the line passing through the origin with a slope $K_r/c$;

ii) choosing an arbitrary Point $O$ as the center of homothety on the line that connects the origin with the point of maximum load $(P_{o}^{max}; y_{o}^{max})$.

iii) plotting Point $P_d$ as the intersection of the line passing through Points $P_s$ and $O$ and the line drawn from the point of maximum load $(P_{o}^{max}; y_{o}^{max})$ with a slope $K_r/c$.

Therefore, the ratio of homothety $d_2/2$, the values of $d_2$ (at the start of loading) and $Al_r$ are determined; Point 8 is easily plotted;

iv) projecting the point of coordinates $P_o = 0$ and $y_o = y_{o}^{max} - d_2$ in the line passing through the point of maximum load $(P_{o}^{max})$ and is parallel to the line $Ps-4$. In this way, Point $P_o$ is established and the value of $P_{o}^{max} - d_2$ is settled;

v) drawing the line $P_d-8$, that is parallel to the line $Ps-4$ and intercepts the horizontal line through the point of maximum load $(P_{o}^{max}; y_{o}^{max})$ in point $P_a$ and so coming to the value of $2A_o(1 - c.d_2/K_r)$ and of $\mu_{o_1}$ using Eq. 31-a;

vi) determining $Q$ iteratively by means of Eqs. 15, 16, 18 and either Eq. 31-a and then the values of $\mu_{o_1}$, $C_1$; and $C_2$; and

vii) comparing the values of $\mu_{o_1}$ of steps v and vi and carrying out another interaction in $\mu$, changing the position of Point $O$, up to convergence.

Application of this procedure was done for the piles of Group II, indicated in Table 2. The results are presented in Table 6 and in Figs. 20 to 25.

For the cases of Cosipa 9 and 10 it was necessary to correct the curves of the rebound, as illustrated in Figs. 21-a and 22-a. This shows the need to get the unloading curve with the same care required for the loading, which does not always happen in practice. Table 5 shows values of $f_o$ of layers 1 and 2 for these cases plus Cosipa 6 and Alamoa-13. For the SFL clays in the Cosipa Area the values of $f_o(1)$ averages 15, agreeing with previous experience (Massad, 2009).

For the pile of Penha there was also a need to correct the curve of the rebound, as illustrated in Fig. 24-a. In this case, the toe reached the maximum resistance $q_t = P_t / S_t$ and $R^* = R \equiv 0!$ This means that resilience is controlled only by the friction reaction. The value of $f_o = 75$ kPa, derived from the data of Tables 2 and 6, coincides practically with the assessment by the already mentioned Décourt-Quaresma Method, i.e., $f_o = 10. (SPT/3 +1) = 10.(18/3+1) = 70$ kPa!

Figures 18 and 25 and Tables 3 and 6 allow comparing the static and dynamic tests on PRE-2 Pile (USP). It should be noted initially that it was necessary to correct the curve of the “rebound” of the Static Load Test, as shown in
in order to “adjust” the value of the elastic rebound ($c_1$), an estimate based on the Van Weele’s Equation indicated in Table 3, related to the Dynamic Load Test in the same pile. Moreover, the values of $c_1$ and $d_2$ adopted for Static Load Test were the same of the Dynamic Load Test. It can be seen that the results were identical. Niyama & Aoki (1991) had already arrived to the same conclusion, in another context.

### Table 6 - Results of the analysis of Group II.

<table>
<thead>
<tr>
<th>Data</th>
<th>Parameter</th>
<th>Cosipa-6</th>
<th>Cosipa-9</th>
<th>Cosipa-10</th>
<th>Alamo 13</th>
<th>Penha P</th>
<th>Pre-2 static</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input</td>
<td>$\mu_{load}$</td>
<td>1.4</td>
<td>1.9</td>
<td>2.0</td>
<td>1.6</td>
<td>1.8</td>
<td>2.0</td>
</tr>
<tr>
<td></td>
<td>$K_r$ (kN/mm)</td>
<td>69</td>
<td>64</td>
<td>83</td>
<td>134</td>
<td>107</td>
<td>333</td>
</tr>
<tr>
<td></td>
<td>$P_{max}$ (kN)</td>
<td>2000</td>
<td>1860</td>
<td>1750</td>
<td>3104</td>
<td>3000</td>
<td>3200</td>
</tr>
<tr>
<td></td>
<td>$\rho$ (mm)</td>
<td>25.5</td>
<td>26.5</td>
<td>23.0</td>
<td>25.0</td>
<td>16.8</td>
<td>8.8</td>
</tr>
<tr>
<td></td>
<td>$P_r$ (kN)</td>
<td>1547</td>
<td>1740</td>
<td>1750</td>
<td>2827</td>
<td>3000</td>
<td>2611</td>
</tr>
<tr>
<td></td>
<td>$y_{rd}$ (mm)</td>
<td>21.7</td>
<td>25.1</td>
<td>23.0</td>
<td>25.0</td>
<td>18.6</td>
<td>6.5</td>
</tr>
<tr>
<td></td>
<td>$d_{m}$ (kN/mm)</td>
<td>10.3</td>
<td>20.8</td>
<td>50.1</td>
<td>16.2</td>
<td>0.0</td>
<td>247.3</td>
</tr>
<tr>
<td></td>
<td>Eq. of line 4-5 $P_o = d_i + d_c y_o$</td>
<td>1388+7.3y_o</td>
<td>1229+20.8y_o</td>
<td>1750</td>
<td>2421+16.2y_o</td>
<td>3000</td>
<td>2398+33.1y_o</td>
</tr>
<tr>
<td></td>
<td>$c$</td>
<td>0.65</td>
<td>0.65</td>
<td>0.50</td>
<td>0.65</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>Output</td>
<td>$\mu A_r$ (kN)</td>
<td>1466</td>
<td>1544</td>
<td>857</td>
<td>2629</td>
<td>3000</td>
<td>1596</td>
</tr>
<tr>
<td></td>
<td>$R S_y$ (kN/mm)</td>
<td>8.2</td>
<td>30.9</td>
<td>0.0</td>
<td>18.5</td>
<td>0.0</td>
<td>36.8</td>
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<tr>
<td></td>
<td>$R' S_y$ (kN/mm)</td>
<td>12.1</td>
<td>30.8</td>
<td>126.4</td>
<td>18.4</td>
<td>0.0</td>
<td>960.8</td>
</tr>
<tr>
<td></td>
<td>$\mu y_1$ (mm)</td>
<td>7.00</td>
<td>6.4</td>
<td>7.1</td>
<td>10.8</td>
<td>4.6</td>
<td>1.1</td>
</tr>
<tr>
<td></td>
<td>$\mu_{as}$</td>
<td>1.76</td>
<td>2.00</td>
<td>2.00</td>
<td>1.78</td>
<td>1.80</td>
<td>2.04</td>
</tr>
<tr>
<td></td>
<td>$k$</td>
<td>3.2</td>
<td>3.8</td>
<td>1.5</td>
<td>1.8</td>
<td>6.1</td>
<td>4.5</td>
</tr>
<tr>
<td></td>
<td>$C_1$ (mm)</td>
<td>19.2</td>
<td>20.0</td>
<td>15.9</td>
<td>15.51</td>
<td>14.0</td>
<td>7.2</td>
</tr>
<tr>
<td></td>
<td>$C_2$ (mm)</td>
<td>6.3</td>
<td>6.5</td>
<td>7.1</td>
<td>9.49</td>
<td>2.8</td>
<td>1.6</td>
</tr>
<tr>
<td></td>
<td>$Q$</td>
<td>1.3</td>
<td>1.9</td>
<td>2.0</td>
<td>1.41</td>
<td>1.1</td>
<td>3.1</td>
</tr>
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</table>

**Figure 20** - Pile COSIPA 6 – Group II. Static loading tests.

Figs. 25-a, in order to “adjust” the value of the elastic rebound ($p$), an estimate based on the Van Weele’s Equation indicated in Table 3, related to the Dynamic Load Test in the same pile. Moreover, the values of $\mu A_r$ and $d_c$ adopted for Static Load Test were the same of the Dynamic Load Test. It can be seen that the results were identical. Niyama & Aoki (1991) had already arrived to the same conclusion, in another context.

### 5.3. Piles of Group III

For the Dynamic Loading Tests with a Single Blow record (Group III), it is assumed that $\mu$ (at the beginning of loading) = $\mu_{as}$ (at the end of rebound) = 2, $B = B_{as}$ and $R = R' = R_{as}$ (Eqs. 20). As a matter of fact, Eqs. 20 correspond to a basic assumption of the Smith Wave Equation Model, as has been described in papers and manuals, for example, GRL (1998) and Rausche (2002). Another assump-
tion is that the shaft load at failure ($A_{f}$) has been fully mobilized.

The following initial parameters are supposed to be known from CAPWAP (Case Pile Wave Analysis Program):

1) $P_{o}^{\text{max}}$, $y_{o}^{\text{max}}$, $K_{p}$ and $\mu A_{p}$, and, therefore, the coordinates of Point $P_{o}$, Eqs. 23 and 24;
2) $R'Sp$, given by:

$$R'Sp = \frac{\text{Toe Load (PDA)}}{\text{Toe Quake}} = y_{z} \quad (32)$$

Figure 21 - Pile COSIPA 9 – Group II. Static loading tests.

Figure 22 - Pile COSIPA 10 – Group II. Static loading tests.

Figure 23 - Pile Alamoa 13 – Group II. Static loading tests.
iii) \( \mu y_o \), taken equal to the average shaft quake; and
iv) the distribution of the load mobilized along the shaft, which allows the estimation of the Leonards and Lovell’s parameter \( c \).

Next, computations are carried out to determine:

a) \( P_o \) and \( y_o \) by means of Eqs. 21 and 22;
b) \( d_{ep} \) given by Eq. 9 \( (R' = R_{mo}) \);
c) \( \rho \) (elastic rebound) using Eq. 17 (Van Weele’s Equation); wanting, the parameters \( C_2 \) and \( C_3 \) may be estimated by means of Eqs. 15 and 16. This step allows to overcome the difficulty of estimating the set \( s \) using the CAPWAP (Uto et al., 1989).

A graphical solution is at hand, because Points \( P_o \) (Eqs. 23 and 24) and \( P_s \) (Eqs. 26 and 27) are known and so the center of homothety, in this case, center of symmetry. The users of CAPWAP have familiarity with this symmetry. From Eqs. 21 and 22 Point 4 is settled and, by symmetry, Point 8.

The long piles K11 and O11 of Table 1 were also driven in Santos Plain, with the subsoil described above. The CAPWAP was applied during pile installation, 3 and 15 days later. The results are presented in Table 7 and in Figs. 26 to 32.

Comparing Figs. 26, 27 and 28 (pile K11) and 29, 30, 31 and 32 (pile O11) it is noticeable that Point 4 is moving to the right as the set up increases. That is, “the hammer did not fully mobilize the pile capacity, notably the toe capac-

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Figure 24 - Pile P (Penha) – Group II. Static loading tests.

Figure 25 - Pile PRE 2 S (USP) – Group II. Static loading tests.

Figure 26 - Dynamic loading test.
ity”, quoting Fellenius (1998), which had arrived to this conclusion earlier, in a similar context.

This statement can be further developed in the light of the results obtained so far. In fact, considering that

\[ \mu A_y = A_y + P_s \]

Eq. 21 may be rewritten as:

\[ P_{\mu i} = A_y + \left(P_s + R'S_y \mu y_1\right) \]  

(33)

where the amount in parenthesis is the mobilized toe reaction up to Point 4. In other words, the toe reacts with the residual load \( P_s = A_y \) (since \( \mu = 2 \)) plus \( R'S_y \mu y_1 \) along the pseudo elastic range of Fig. 5-b. Figures 33-a and b show how the two parcels of Eq. 33 vary with the time of restrike.

For both piles Fig. 33-a reveals that the total shaft friction increases with time due to set up effects. But the mobilization of the toe reaction is different. Reporting to Fig. 33-b and Table 7, for Pile K11 the toe reaction amounts 8 to 9 MPa, as the set (s) decreases from 6 to 4 mm; and for Pile O11 it increases from 2 to 9 as the set (s) varies from 10 to 4 mm.

Table 7 - Results of the analysis of Group III.

<table>
<thead>
<tr>
<th>Data</th>
<th>Parameter</th>
<th>Pile K11</th>
<th>Pile O11</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Installation</td>
<td>Restrike</td>
<td>Restrike</td>
</tr>
<tr>
<td></td>
<td>(*)</td>
<td>3 days (*)</td>
<td>15 days (**)</td>
</tr>
<tr>
<td>Input</td>
<td>( \mu )</td>
<td>2.00</td>
<td>2.00</td>
</tr>
<tr>
<td></td>
<td>( \mu_{\text{in}} )</td>
<td>2.00</td>
<td>2.00</td>
</tr>
<tr>
<td></td>
<td>( K_{\text{i}} ) (kN/mm)</td>
<td>223</td>
<td>223</td>
</tr>
<tr>
<td></td>
<td>( P_{\text{max}} ) (kN)</td>
<td>7825</td>
<td>6538</td>
</tr>
<tr>
<td></td>
<td>( \mu A_y ) (kN)</td>
<td>2614</td>
<td>3627</td>
</tr>
<tr>
<td></td>
<td>( R'S_y ) (kN/mm)</td>
<td>572.7</td>
<td>485.3</td>
</tr>
<tr>
<td></td>
<td>( \mu y_1 ) (mm)</td>
<td>5.00</td>
<td>4.80</td>
</tr>
<tr>
<td></td>
<td>( c )</td>
<td>0.73</td>
<td>0.65</td>
</tr>
<tr>
<td>Output</td>
<td>( P_{s} ) (kN)</td>
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<td>5957</td>
</tr>
<tr>
<td></td>
<td>( y_{s} ) (mm)</td>
<td>26.4</td>
<td>25.9</td>
</tr>
<tr>
<td></td>
<td>( k )</td>
<td>2.3</td>
<td>3.4</td>
</tr>
<tr>
<td></td>
<td>( d_{s} ) (kN/mm)</td>
<td>160.3</td>
<td>152.6</td>
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<tr>
<td></td>
<td>( \rho ) (mm)</td>
<td>41.0</td>
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</tr>
<tr>
<td></td>
<td>( s ) (mm)</td>
<td>6</td>
<td>0.1</td>
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<tr>
<td></td>
<td>( C_{\text{f}} ) (mm)</td>
<td>31.9</td>
<td>23.7</td>
</tr>
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<td></td>
<td>( C_{\text{i}} ) (mm)</td>
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<td>6.0</td>
</tr>
<tr>
<td></td>
<td>( Q )</td>
<td>3.6</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Notes: (*) Hydraulic Hammer BSP-CG 240, (**) Hydraulic Hammer HH-JUNTTAN.

Figure 27 - Dynamic loading test.

Figure 28 - Dynamic loading test.
In these cases, the hammer did mobilize the toe capacity, perhaps not fully. Ghilardi & Massad (2006) found values from static load tests ranging from 8 to 14 MPa for piles embedded in residual soils in Santos Plain. Note that the transferred energy (Fig. 34) remained almost constant for 3 and 5 days restriking of Pile K11. Figure 35 shows that the maximum displacement \( (\rho + s) \) reached a constant value of 30 mm, confirming that at 15 days of restriking the piles reached a firm substratum.

Figure 29 - Dynamic loading test.

Figure 30 - Dynamic loading test.

Figure 31 - Dynamic loading test.

Figure 32 - Dynamic loading test.

Figure 33 - Separation of loads during restriking.

Figure 34 - Transferred energy and time to restrike.
Other results are shown in Table 8. The mobilized unit shaft friction ($f$) of the SFL clays were plotted in Fig. 36, taking $\mu = 2$. In this figure the dash-dot line represents the value obtained in an instrumented bitumen coated pile subjected to static loading test in Santos Plain, 130 days after installation. Considering that the Piles K11 and O11 were also coated with bitumen, the agreement is reasonable due to expected scatter of values. The scatter of $f$ values in Santos Plain is shown in Fig. 36: the dashed lines represent the upper and lower bounds of the mobilized unit shaft friction ($f$) derived from static loading tests in piles without bitumen. These results validate the assumption made before that the shaft load at failure ($A_s$) has been fully mobilized.

6. Conclusions

This paper showed that a resilient condition may be reached during pile driving or under static cyclic and even monotonic loading. This fact was confirmed by Van Weele’s data on an instrumented precast concrete pile subjected to static cyclic loading test. Under this condition, the loading and unloading movement curves at pile head are homothetic.
The notable points of the homothety were set based on a mathematical model that incorporates, as load transfer functions, the modified Cambefort’s Relations, considers pile compressibility, the residual stresses and matches the Cambefort parameters for loading and unloading (rebound).

A graphical construction was developed along with a numerical procedure – the Homothetic Model - allowing determining the notable points together with significant parameters of pile-soil interaction like the unit shaft friction, the toe stiffness and resistance besides the shaft and toe quakes. In particular, the mobilization of shaft and toe plus residual loads was clarified with the concept of resilience.

The application of the Homothetic Model to several piles allowed improving the understanding of their behavior under static or driving loadings. In particular, a new procedure was developed to analyze dynamic loading tests with a single blow record, considering the residual load at the toe. Underlying this approach is the need to get the unloading curve with the same care required for the loading, which does not always happen in practice.

Acknowledgments

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References


List of Symbols

- $A$: Total lateral (shaft) load
- $A_r$: Total lateral (shaft) load at failure
- $A'$: Average value of lateral (shaft) load
- $B$: Caberfort Parameters (see Fig. 5-a)
- $c$: Ratio of the average value of the transferred lateral load and $A_r$ (see Eq. 4)
- $C_s$: shaft quake
- $C_t$: Toe quake
- $d_s$: Stiffness of the set pile-toe soil
- $D$: Diameter of solid pile
- $D_s$ and $D_i$: Outside and inside pile diameters
- $E$: Modulus of elasticity of the pile
- $f$: Unit skin friction
- $f_u$: Maximum (ultimate) unit skin friction
- $f_r$: Residual unit skin friction
- $G_s$: Shear modulus of the soil at the pile toe
- $h$: Pile length embedded in soil
- $h_1$: Width of upper layer of the subsoil
- $k$: Relative stiffness of the pile-soil (shaft)
- $K_s$: Pile stiffness, as a structural piece
- $L$: Dimension of a square pile
- $P_r$: Residual toe load
- $P_{reb}^y$: Residual toe load at the end of rebound
- $P_v$: Vertical load at the pile head
- $P_{oc}$: $P_r$ associated to full lateral friction mobilization
- $P_{oc}$: $P_r$: Notable points of Fig. 8.
- $P_{o_2}$ or $P_{o_1}$: Maximum value of $P_o$
- $q_r$: Toe pressure
- $Q$: $C_t/l_y$ (Eq. 18)
- $Q_r$: Toe load
- $Q_{max}$: Maximum value of $Q_r$
- $r$: See Eq. 31-c
- $R$: Soil stiffness at the pile toe (see Fig. 5-b)
- $s$: Pile set
- $S$: Cross sectional area of the pile shaft
- $S_t$: Cross sectional area of the pile toe
- $SPT$: Standard Penetration Test blow count
- $y$: Pile movement
- $y_1$, $y_2$, $y_{3r}$: Movements of the pile at head and bottom
- $z$: Square root of $k$
- $\Delta e$: Pile shortening or lengthening
- $\mu$: Magnifier factor of shaft load due to $P_s$
- $v$: Poisson’s ratio
- $\rho$: Elastic rebound measured at the pile head